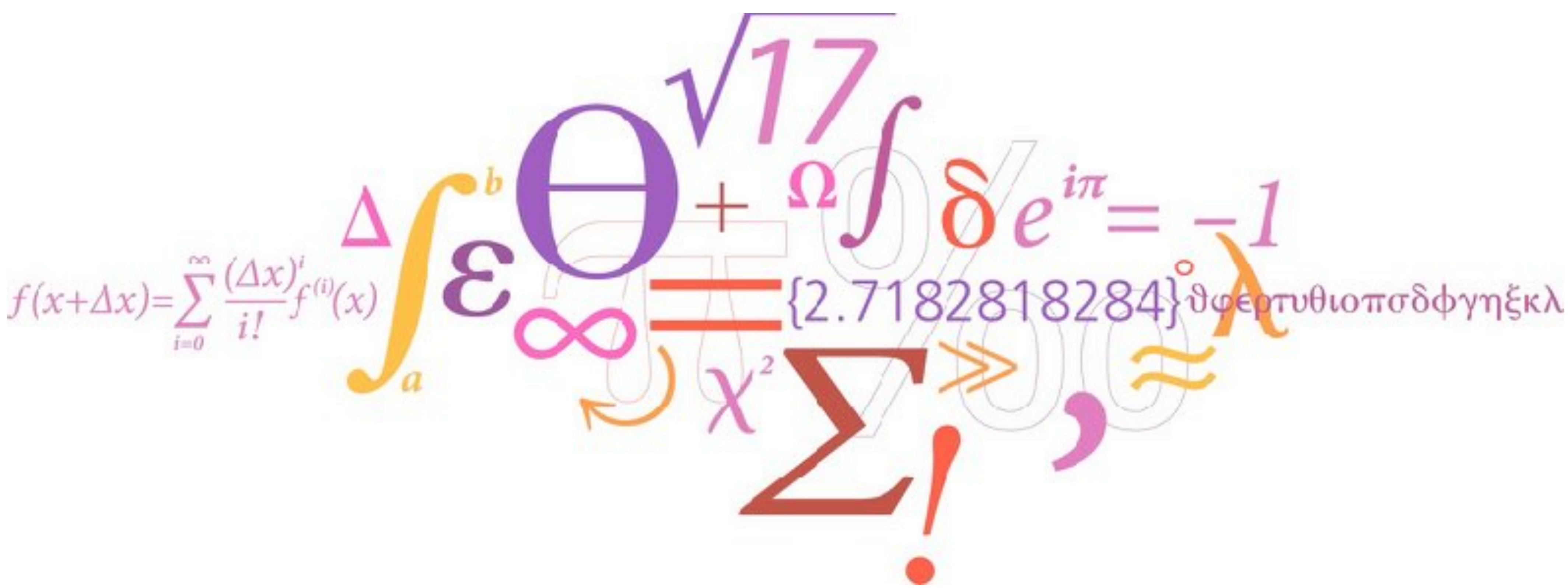
Mathematical Study Groups with Industry



Rostock Nov 30 2016

P. G. Hjorth
Department of Applied Mathematics
and Computer Science
Technical University of Denmark

What is

a

Mathematical Study Group with Industry ?

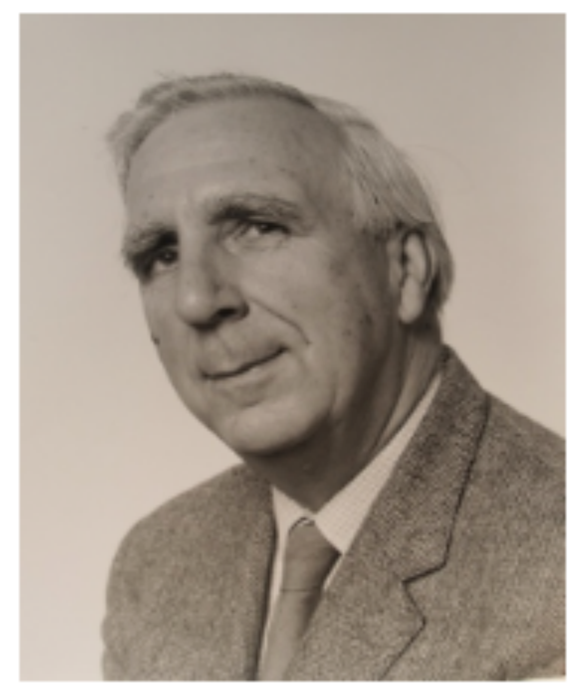
A Mathematical Study Group with Industry
is **workshop** where researchers from industry
submit **real-world mathematical problems**,
and then work with academic mathematicians
to solve those problems.

HISTORY

Study Groups began in Oxford, UK

In 1968, two Oxford applied mathematicians, **Alan Taylor** and **Leslie Fox**, organised the first meeting of this kind, inviting industry contacts to challenge university mathematicians with problems of modelling or computation.

Alan Tayler



Leslie Fox

The meetings became annual events in the 1970's and the 1980's.

They began to be conducted at other UK universities. In the 1990's Study Groups began to be organised at universities outside the UK.

In agreement with **ECMI** (European Consortium for Mathematics in Industry), the meetings were referenced as ESGIs (European Study Groups with Industry), and enumerated.

The number of Study Groups conducted in various European contries has grown rapidly; and in spring 2014 the 100'th Study Group was conducted, (very appropriatly in Oxford). For upcoming study groups, see, e.g.,

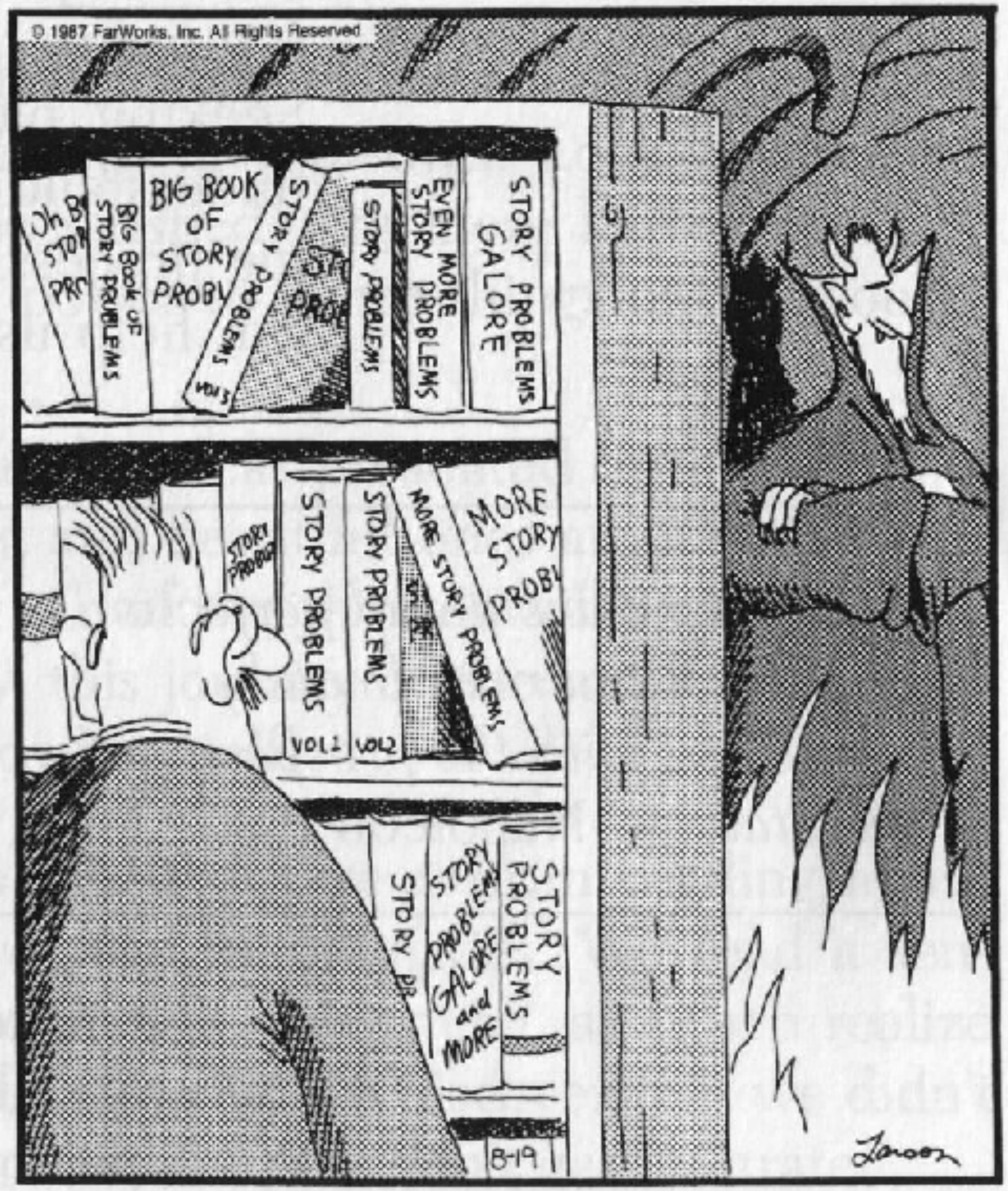
https://ecmiindmath.org/study-groups/

WHY

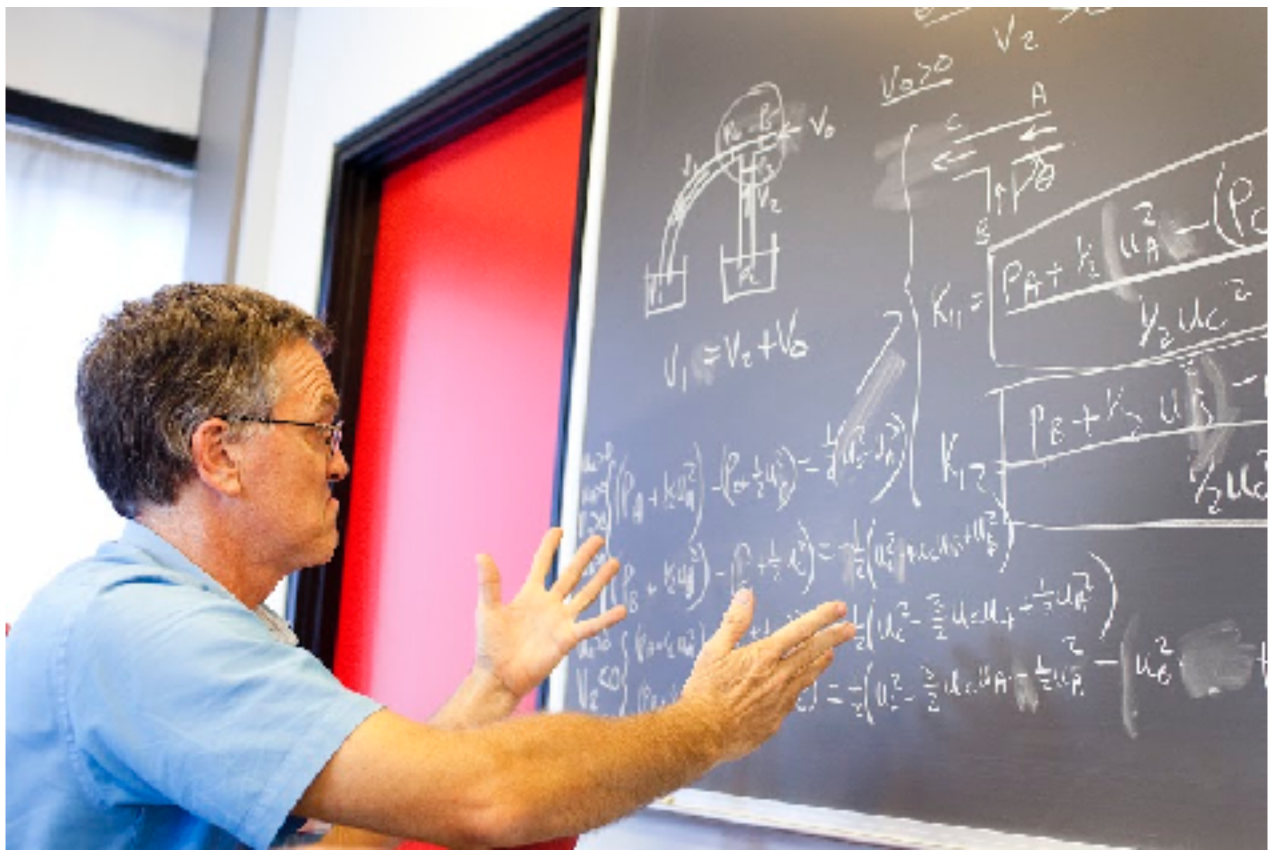
STUDY GROUPS

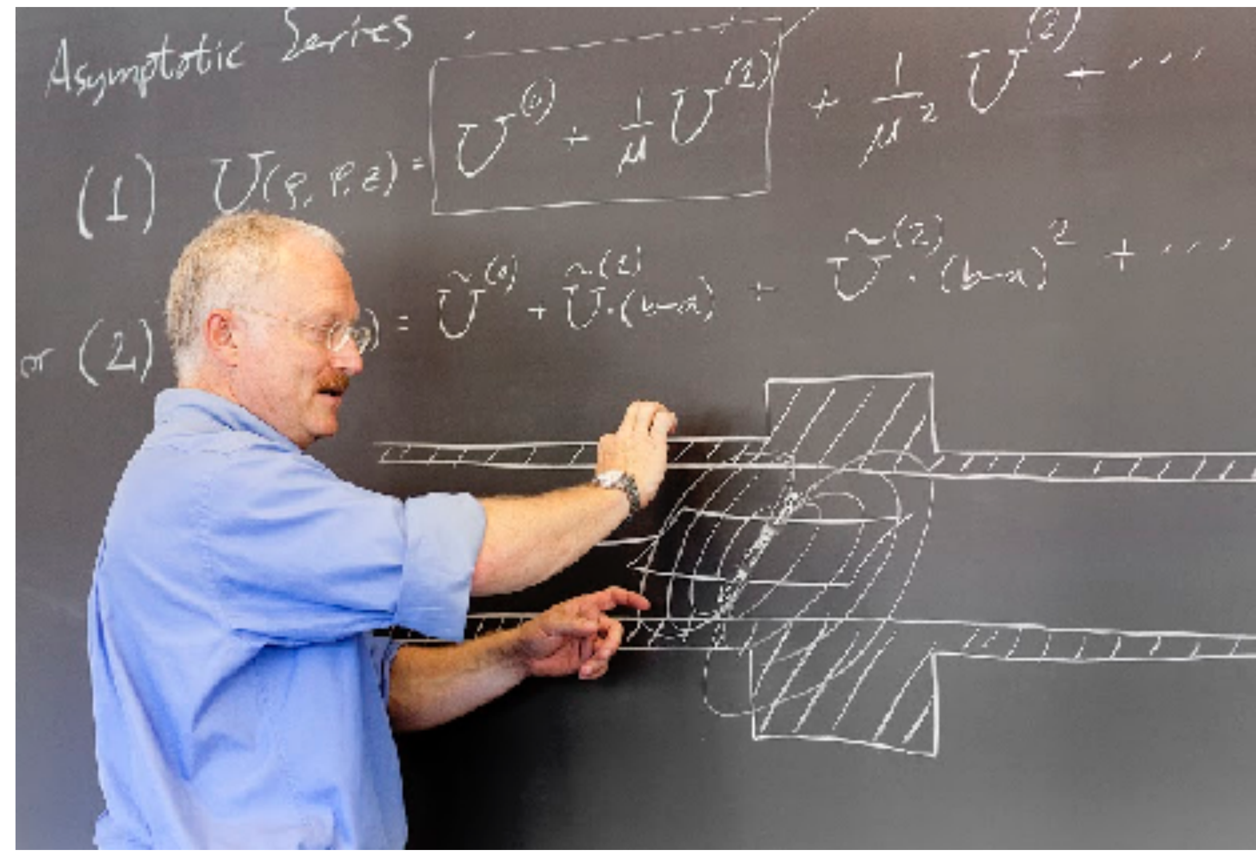
?

THE FAR SIDE By GARY LARSON



Hell's library





consider

a spherical cow

Win/Win

Companies:

Get their problems ‘flash fried’ in one week by intensive work by independent experts

Establish contact with academics, as well as students (potential recruits)

Academics:

Test their skills on real-world problems. Get new ideas for research and teaching.

Establish contact to companies, where students can do projects.

Format of a typical ESGI week:

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
	Problem Presentations	Work	Work	Work	Group Presentations
Participants arrive	Groups form Work begins	Work	Work Progress Summary	Work Report prep	

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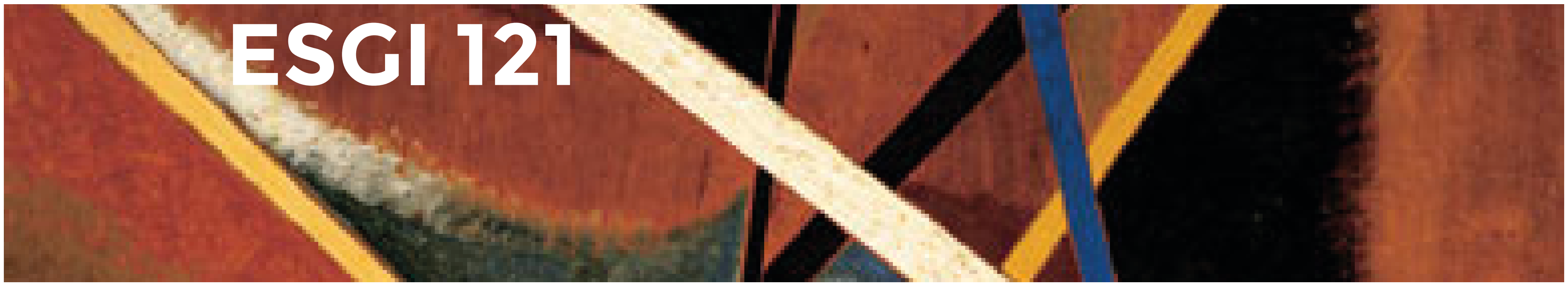
"Modern Methods of Industrial Mathematics"
DTU course 02940: 5 ECTS

+ coursework for phd students



EUROPEAN STUDY GROUP with INDUSTRY

TECHNICAL UNIVERSITY of DENMARK August 15 - 19 , 2016

11. + 12. + 13. August:

phd course "Modern Methods in Industrial Mathematics"

125th European Study Group with Industry (ESGI)

1st Study Group with Industry in Cyprus

5-9th December 2016, Limassol, Cyprus

Committees

Local Organising Committee

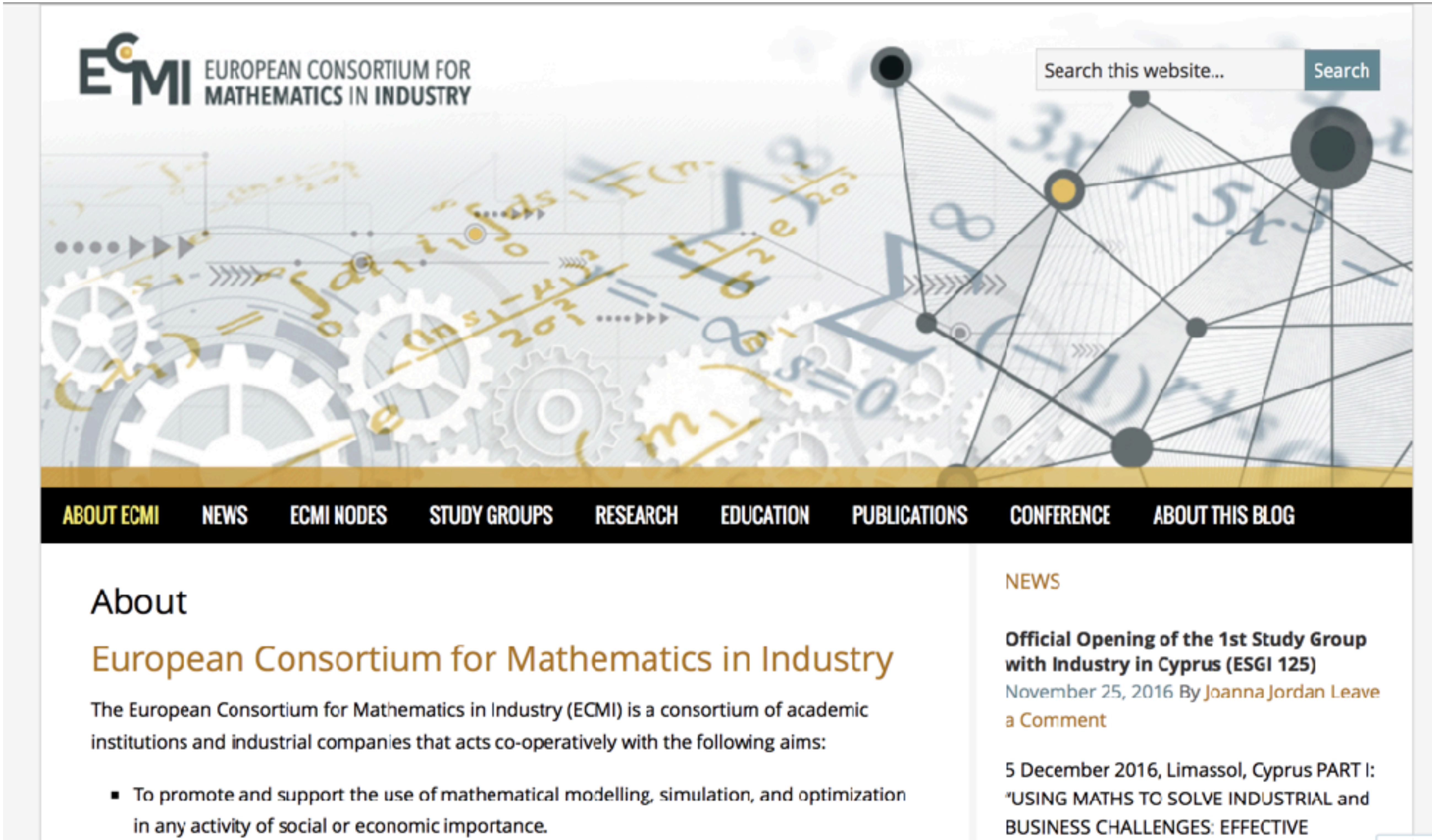
- Dr Katerina Kaouri, Cyprus University of Technology (Chair, MI-NET member)
- Dr Margarita Zachariou, Cyprus Institute of Neurology and Genetics (Co-Chair, MI-NET member)
- Dr Paul Christodoulides, Cyprus University of Technology
- Dr Andreas Kyprianou, University of Cyprus

Scientific Board

(The Scientific Board consists of distinguished experts who assist the Local Organising Committee in selecting the Study Group problems and also provide expertise to the teams during the Study Group as necessary):

- Dr David Allwright, Smith Institute for Industrial Mathematics and Systems Engineering (University of Oxford), UK
- Prof. Georgios Georgiou, Department of Mathematics and Statistics, University of Cyprus
- Prof. Christoforos Hadjicostis, Department of Electrical and Computer Engineering, University of Cyprus (Dean of the Engineering School)
- Prof. Poul Hjorth, Department of Applied Mathematics and Computer Science, Technical University of Denmark, Denmark (Executive Director of the European Consortium for Mathematics in Industry (ECMI))

www.ecmiindmath.rod



DTU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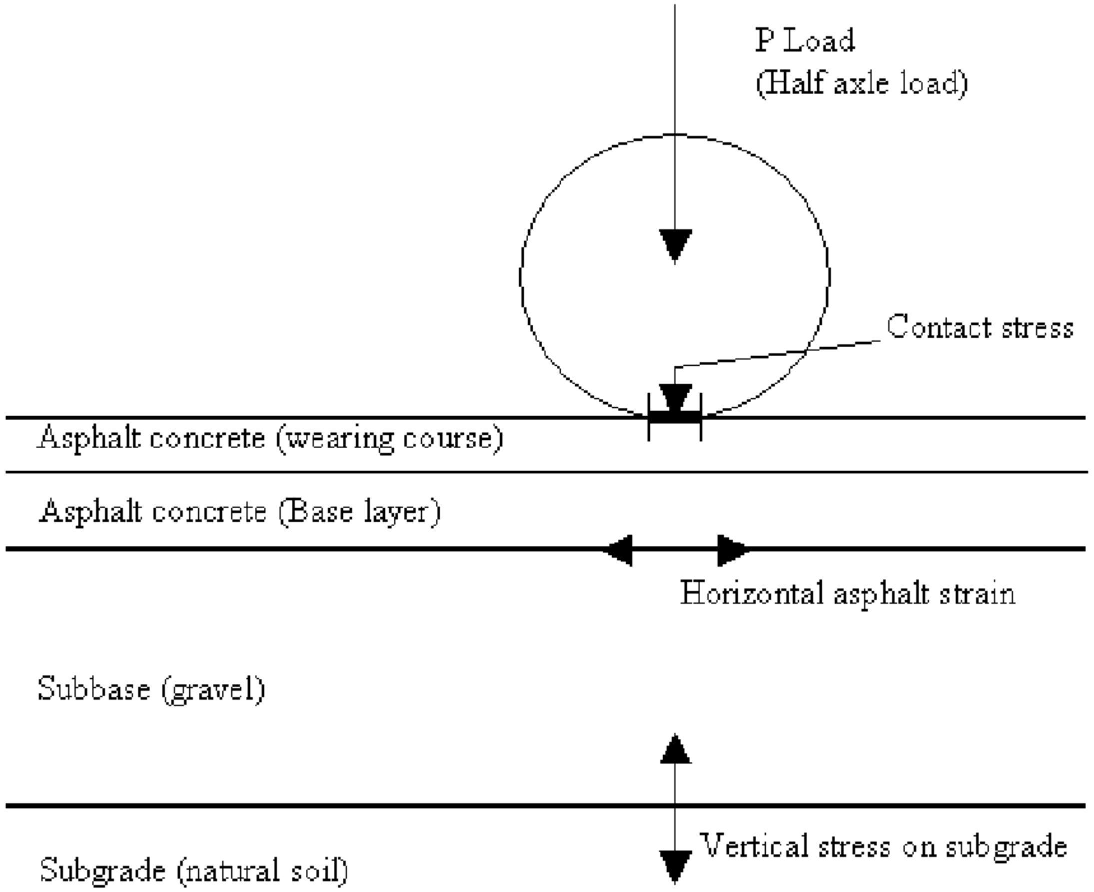
Examples of Problems

- Greenwood Engineering
- LEGO: How to build with LEGO
- NOVO: design of pharmacophores
- DSB S-train planning
- Scroll Compressor & Moineau Pump
- BAE Systems: WLAN frequencies
- Penguin Eggs

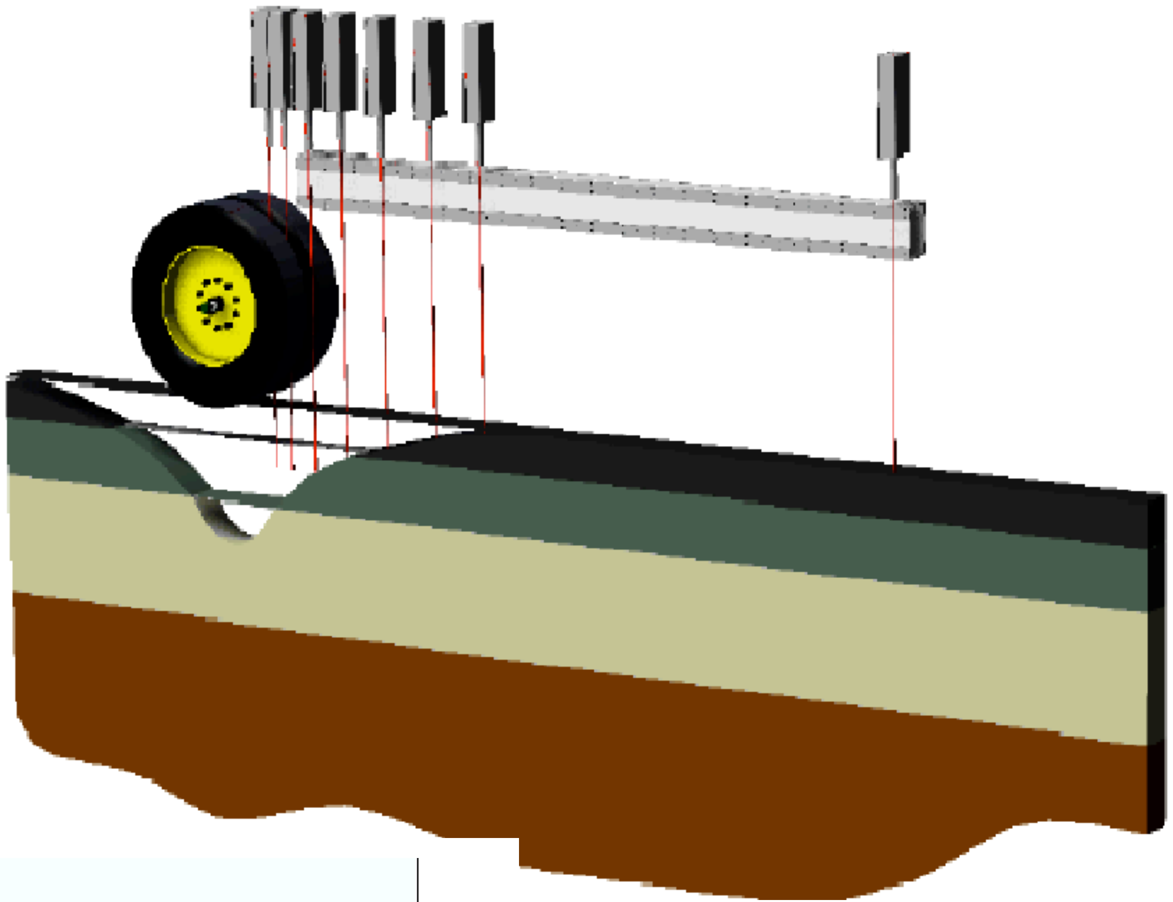
Greenwood Engineering:

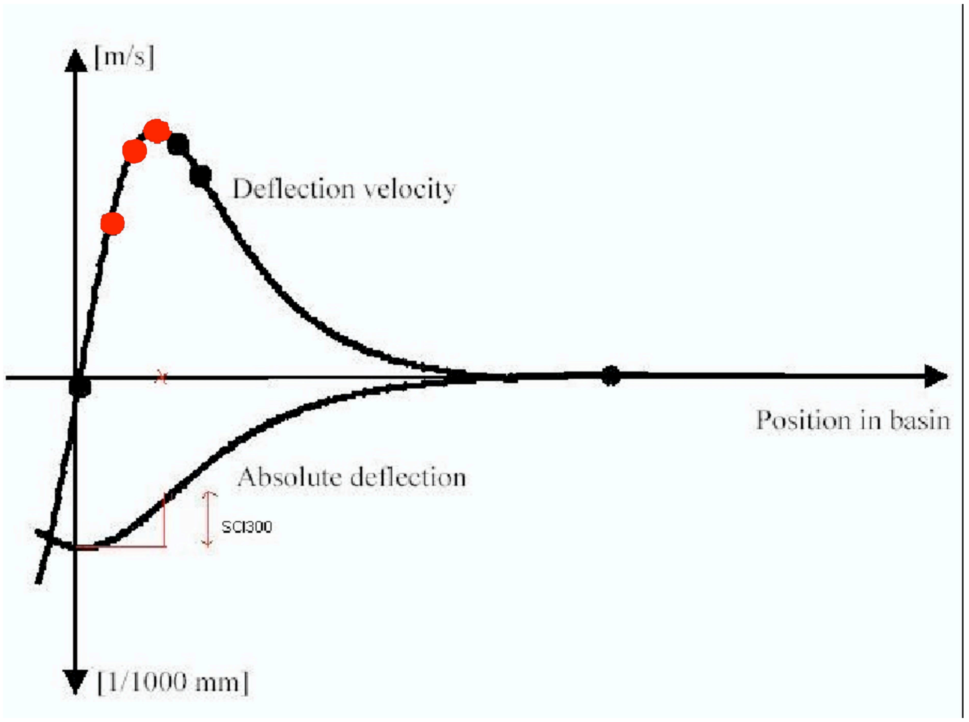
Bearing Capacity of Roads



The condition and durability of an asphalt road can be estimated from the deflection caused by a given load.

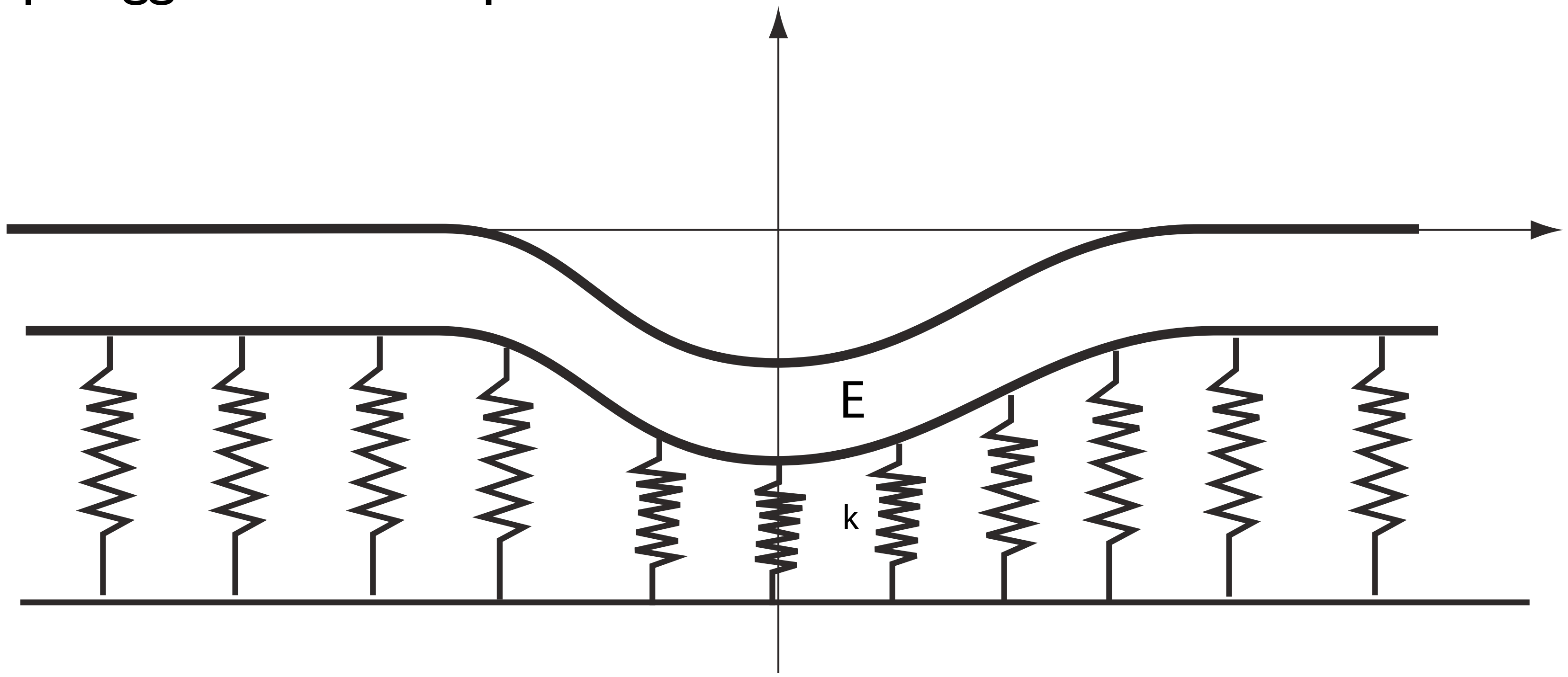
The principle of Greenwood Engineerings Highspeed Deflectometer is to measure deflection velocities using doubler lasers (presently only three).





Greenwood asked for an extrapolation method based on a mechanical model of the road.

The Study Group suggested a two-parameter model:

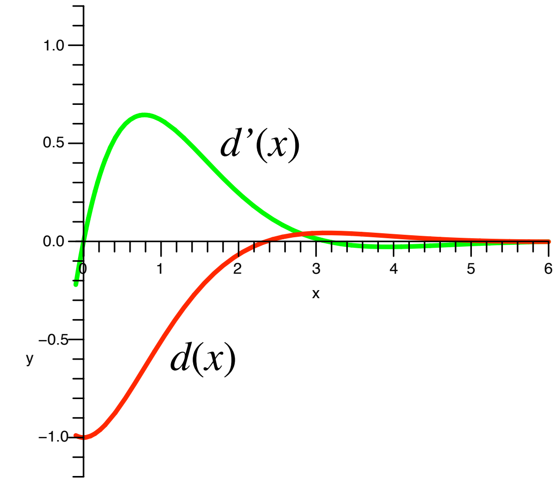


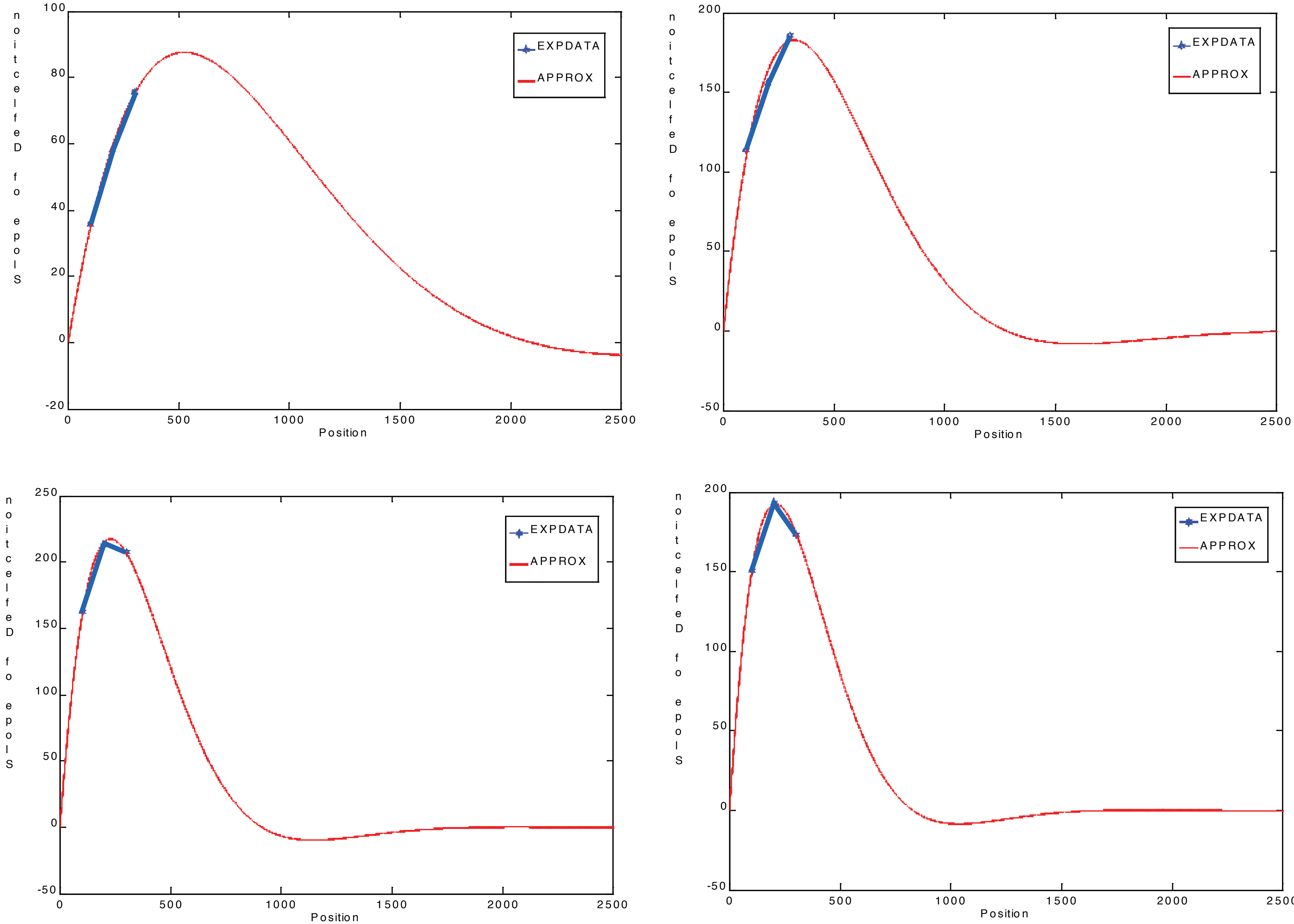
With a point load at $x = 0$, the resulting beam equation is

$$\frac{\mathrm{d}^4 d(x)}{\mathrm{d}x^4} = -\frac{F}{E}\delta(x) + \frac{k}{E}d(x).$$

This results in a two-parameter $(A, B > 0)$ family of solutions for the displacement $d(x)$:

$$d(x) = -\frac{A}{2B}e^{-Bx}(\cos(Bx) + \sin(Bx))$$





Fit of the Winkler model to four set of measurements.

LEGO:
how to build with Bricks

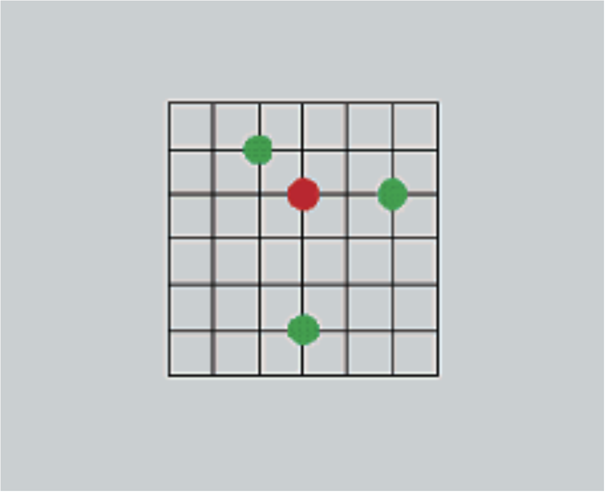
LEGO asked for algorithms to output optimal building instructions for arbitrary 3D design shapes.

The Study Group provided:

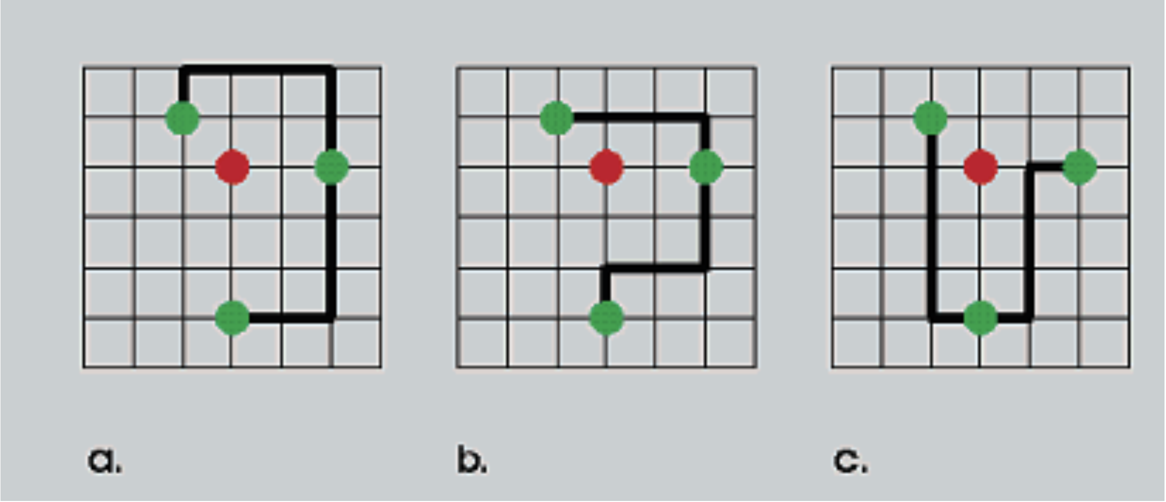
Optimal Building algorithms for simple shapes: boxes, sheets, and beams.

Arguments why the general optimal problem is extremly difficult (possibly np).

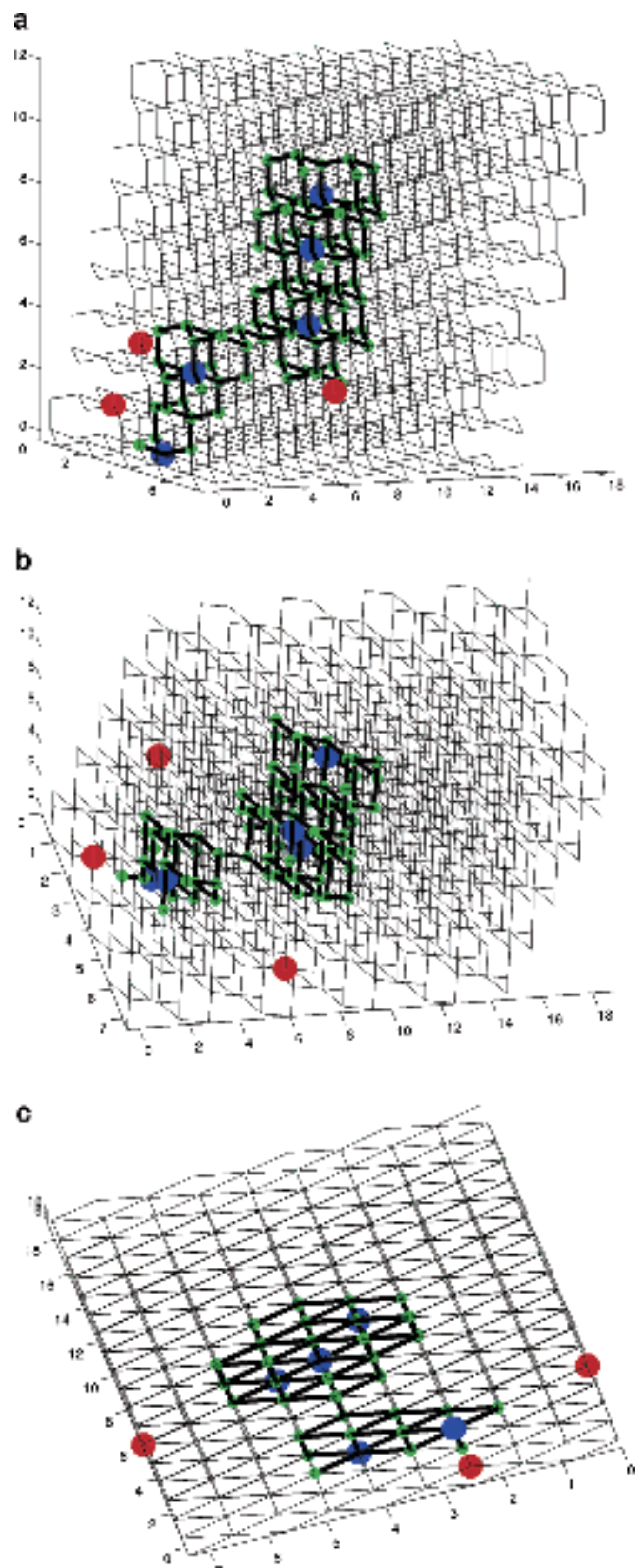
NOVO: Design of Pharmacophores



Systematic molecular design = find connected lattice graphs having vertices at selected points and no edges through anti-points.



Examples



NOVO asked for a systematic description based on a *diamond lattice* of graphs that contain nodes and anti-nodes (possibly weighted), as well as a measure of the 'goodness' of the solutions.

The ESGI group employed methods from *Topology Optimization*. The graphs are generated iterativly by a computer code which takes into account known nodes and anti-nodes, and optimizes simple functionals.

The algorithm was implemented in MATLAB and tested on known examples. It worked.

See J. Chem. Inf. Comput. Sci. **2004**, 44, 856-861

DSB: S-trains

Work schedule for S-train drivers

The situation prior to ESGI:

Efficency
= (time driving the train)/(total time at work) ≈ 60%

Robustness ≈ 2 minutes (if a driver is more than two minutes late for his break, he is not allowed to drive the next train).

The situation after ESGI:

Efficient increased up to 83 % (without violating union rules).

A driver can stay on the same line with a "standard-plan" which fits 73% of all departures.

This "standard plan" has an efficiency of 80 %.

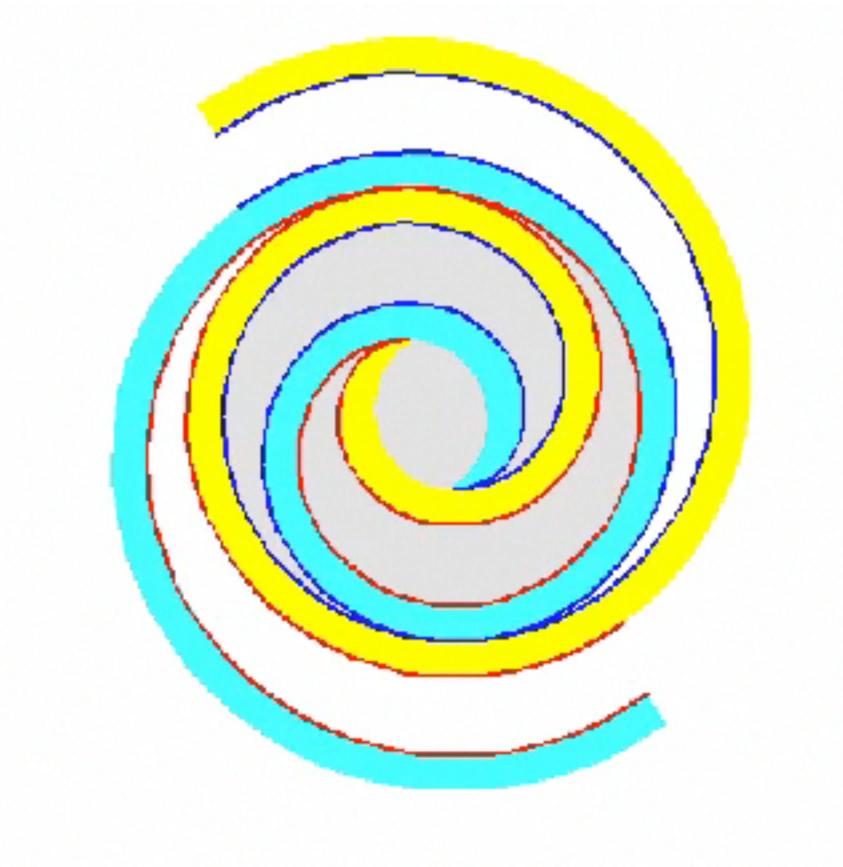
The "standard plan" has a robustness of 9 minutes.

The rubustness can be increased to 13 minutes with a slightly lower effciancy.

The phd student working on the problem was later hired by S-train company.

Danfoss: Scroll Compressor

Patent by Leon Crux (1905)

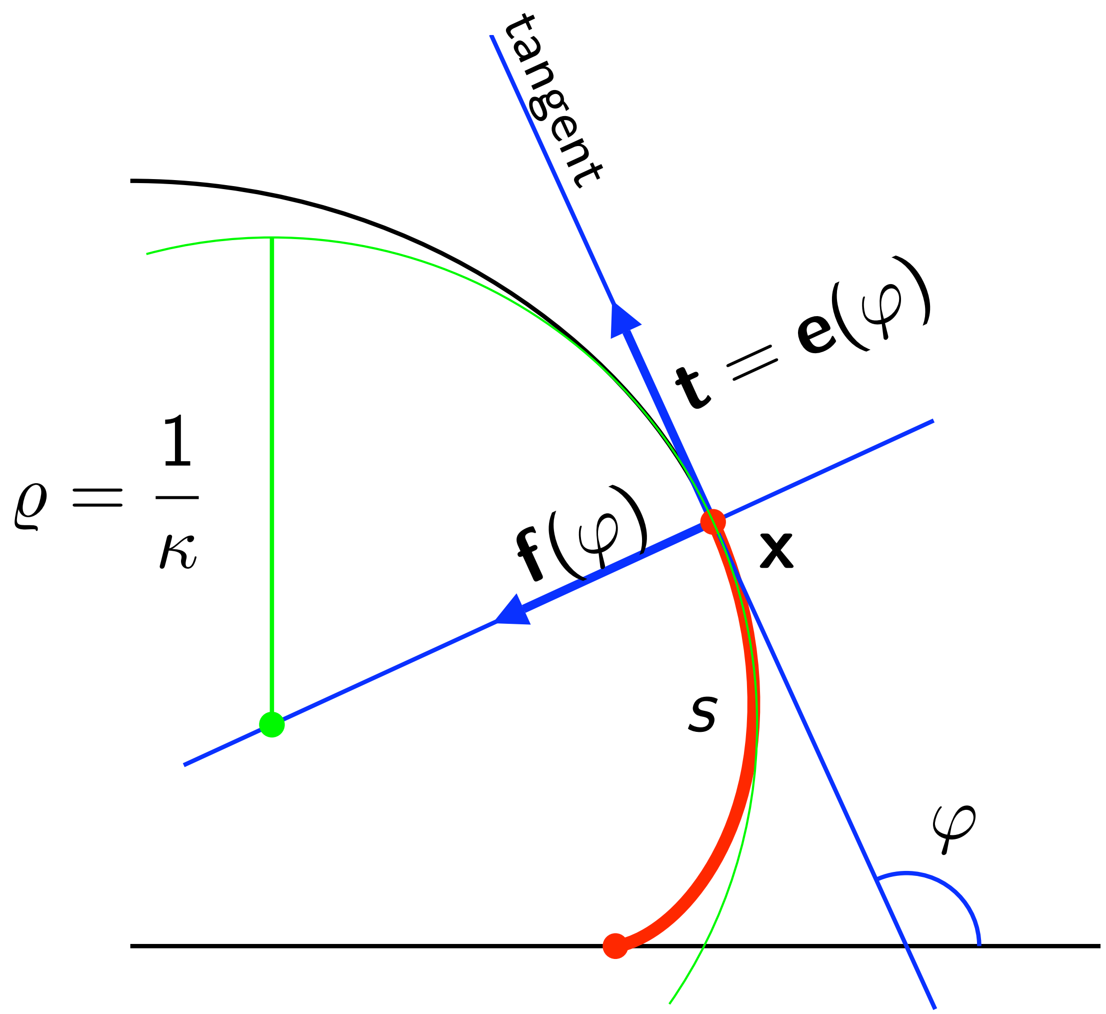


Danfoss asked for a mathematical description that would facilitate design experimentation.

In particular higher compression was needed for use in refrigirators and freezers

Answer

Use tangent direction as function of arc length to describe the wall of the moving spiral.



$$\mathbf{e}(\varphi) = (\cos(\varphi), \sin(\varphi))$$
$$\mathbf{f}(\varphi) = (-\sin(\varphi), \cos(\varphi))$$

φ is monotone and can be used as parameter.

Radius of curvature as a function of φ determines the entire geometry of the scroll compressor.

Wall thickness, volume of compression chamber can be shown to be polynomial expressions in φ, leading to closed analytical expressions.

A 2004 Danfoss prototype:

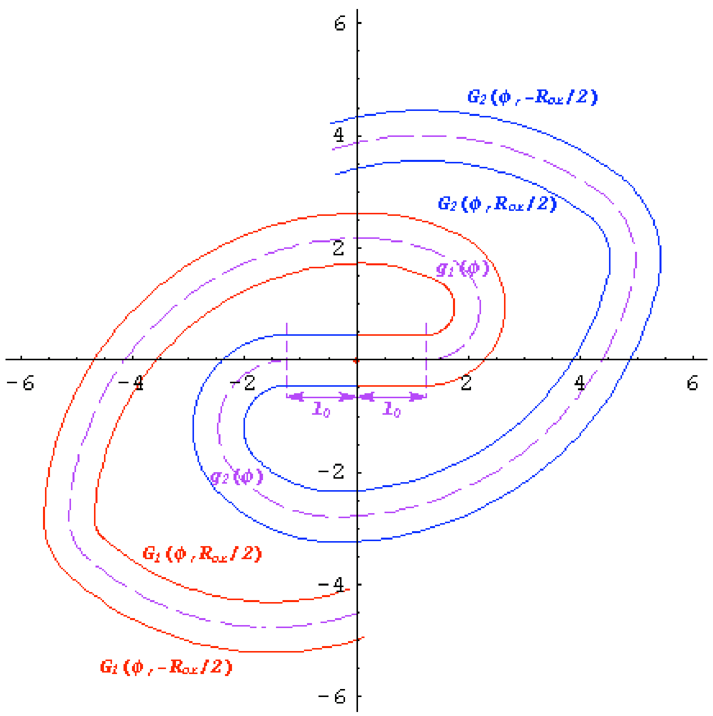


Figure 3 Geometry of the prototype compressor

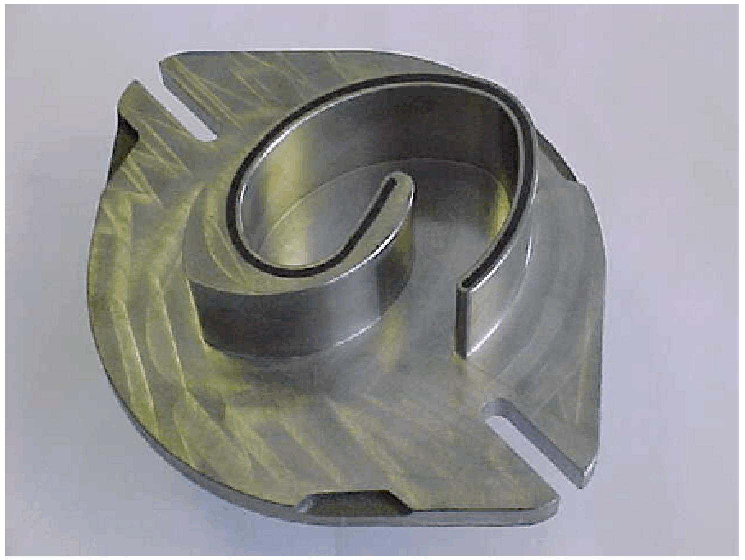


Figure 4 Moving scroll V1.0

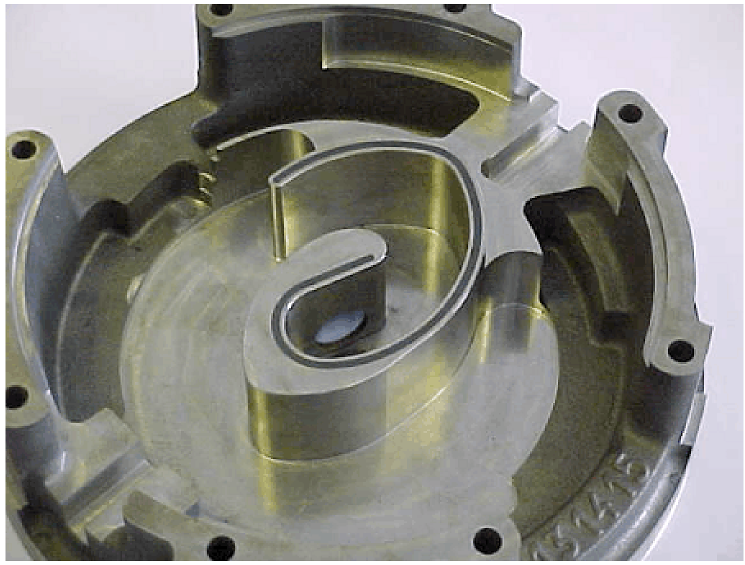
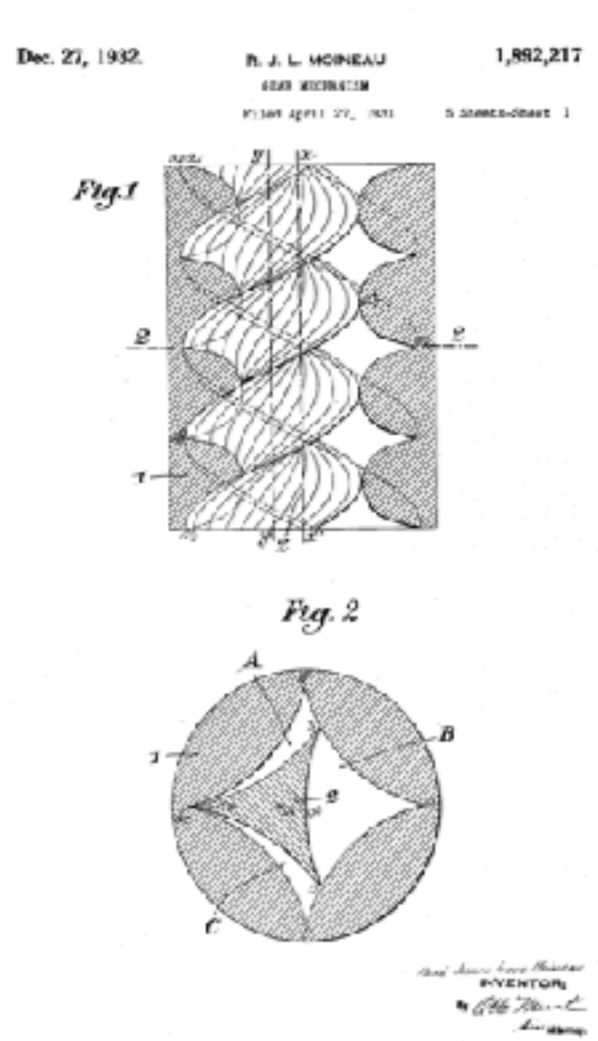


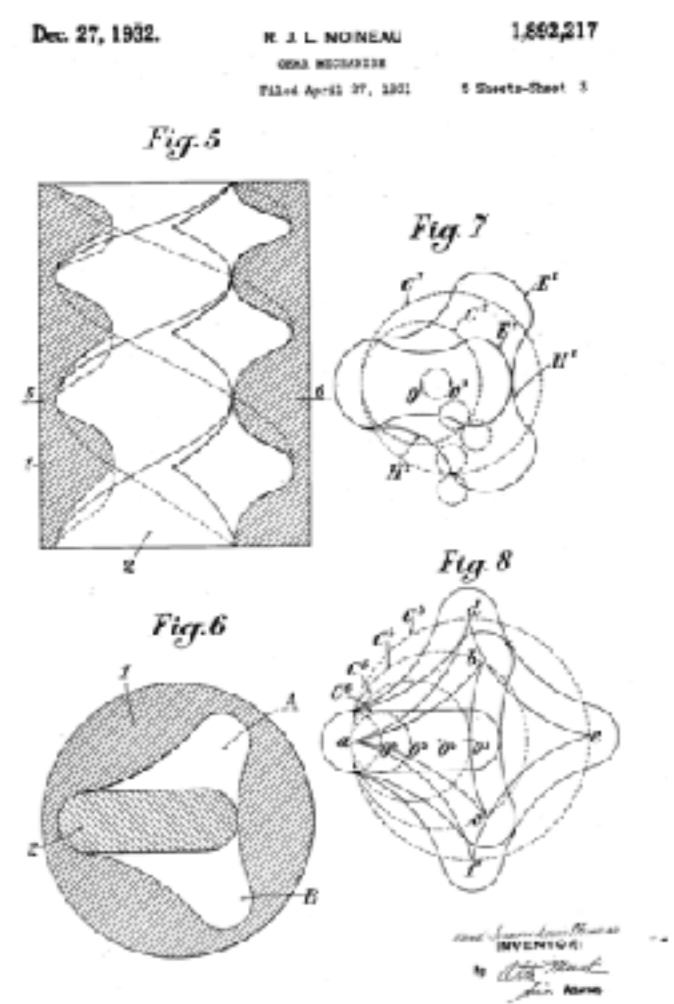
Figure 5 Fixed scroll V1.0

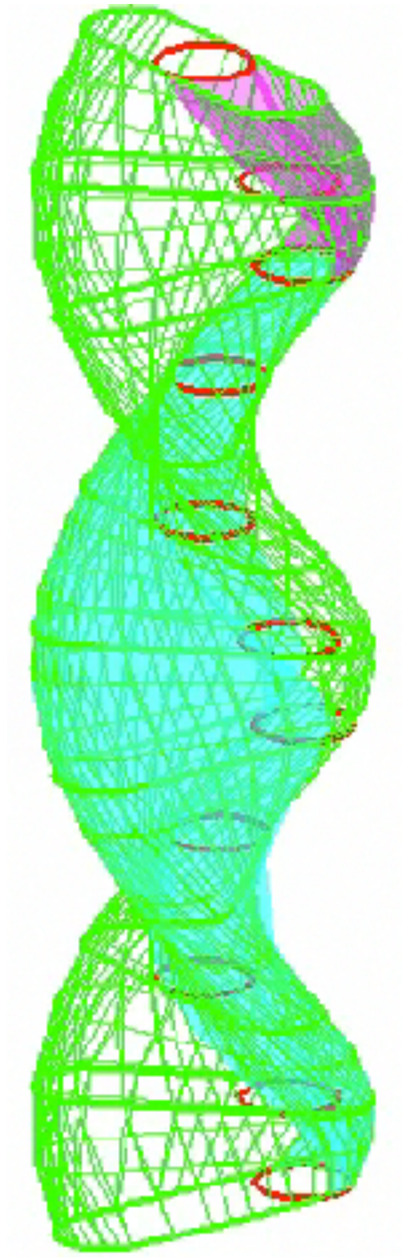
See: SIAM Review 43, pp 113-126 (2001)

Grundfos: Moineau Pumps

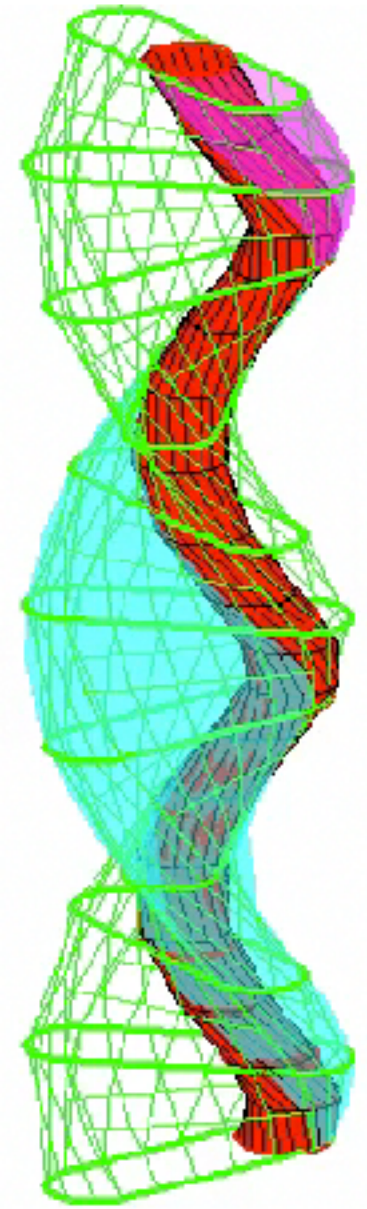
Patent by J. L. Moineau (1932)



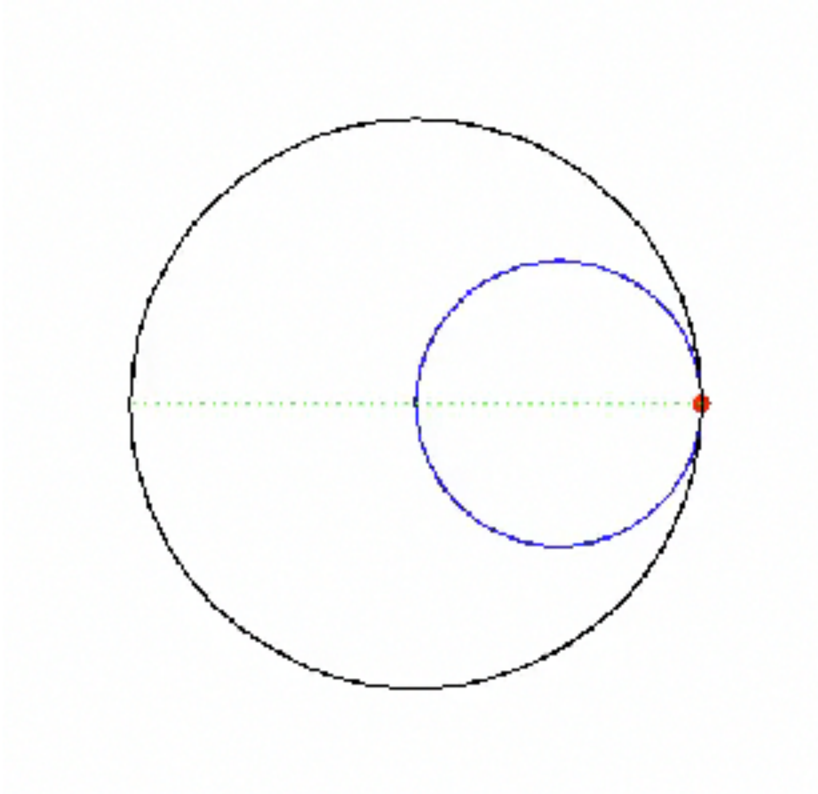


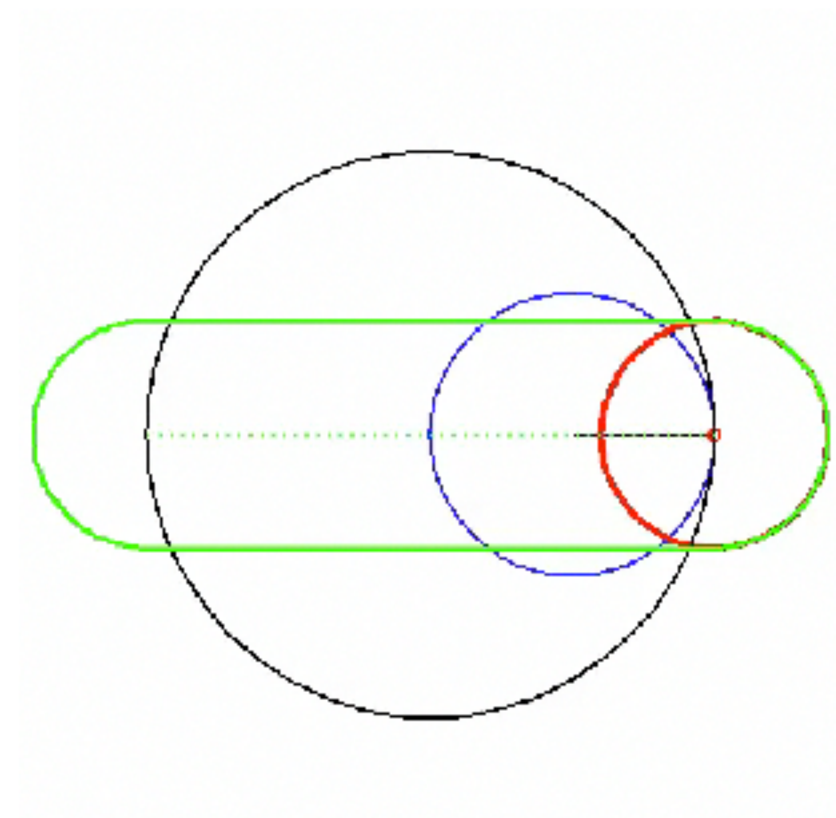


A series of chambers
of constant volume are
pushed up (or down)
the pump chamber by
the contimous motion
of the excentrically
moving pump shaft.

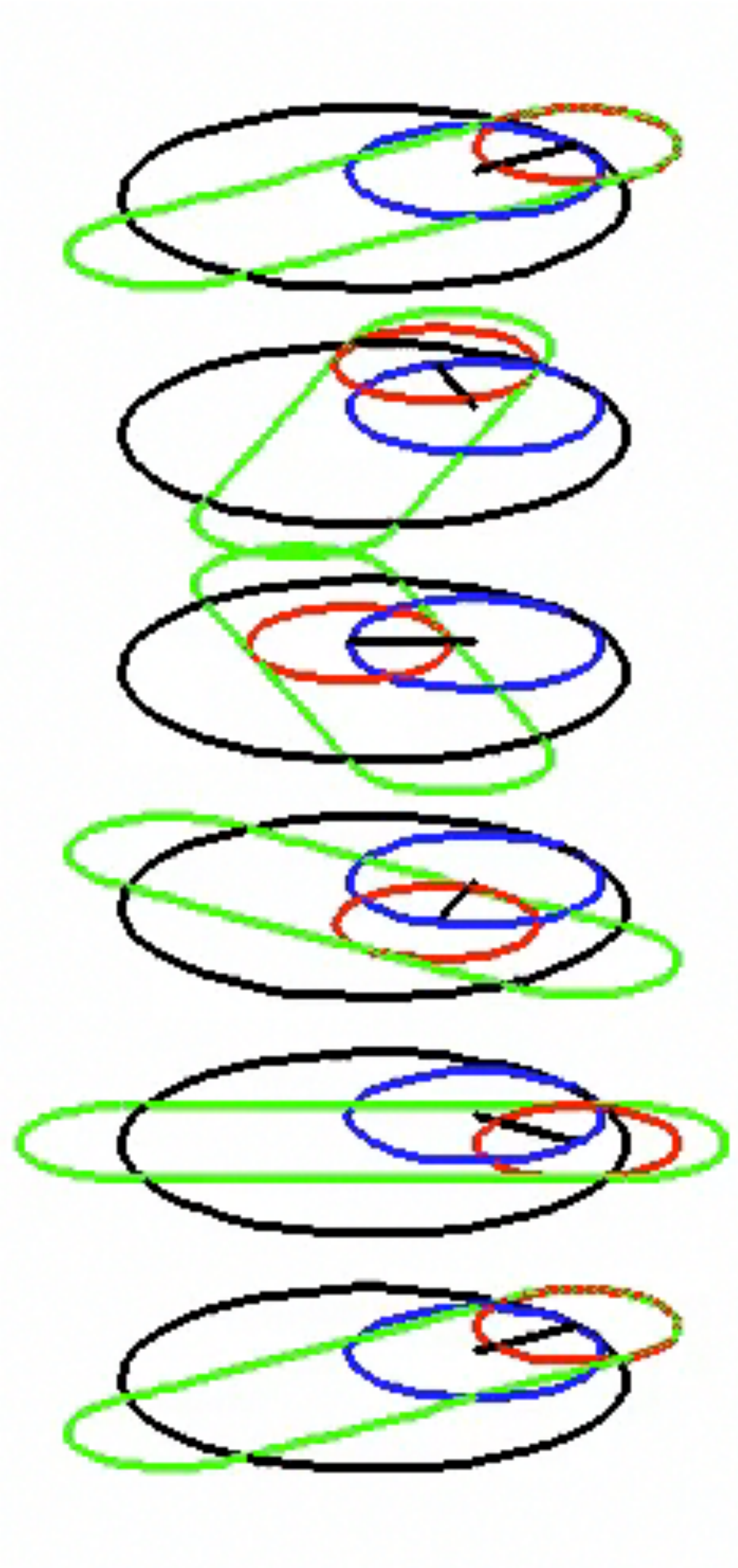


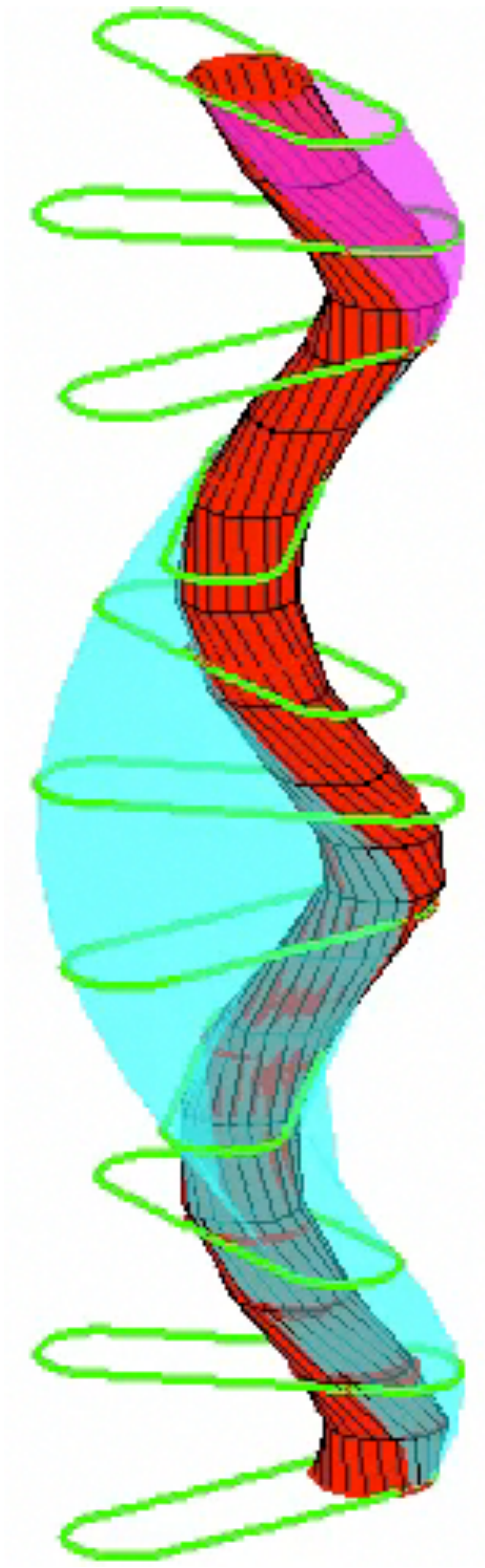
Rolling a circle of radius one on the inside of a circle of radius two will make a point fixed on the inner circle trace out a diameter of the outer circle



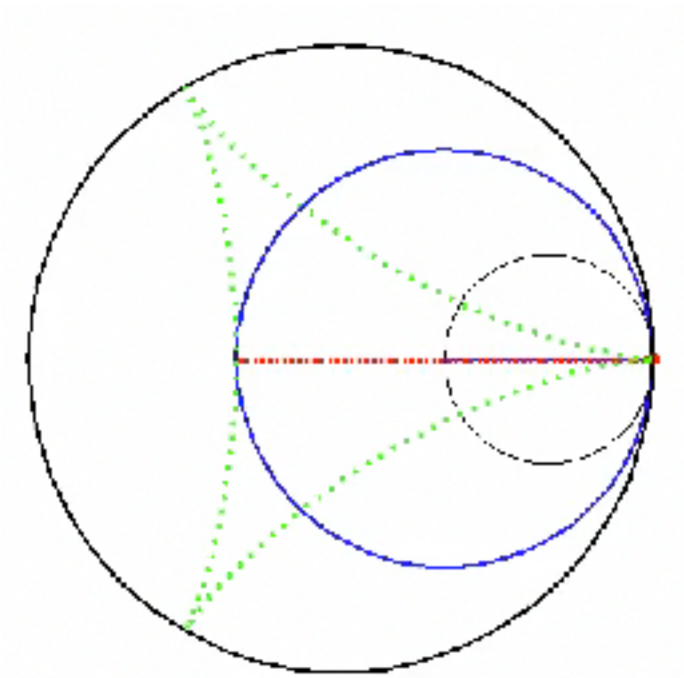


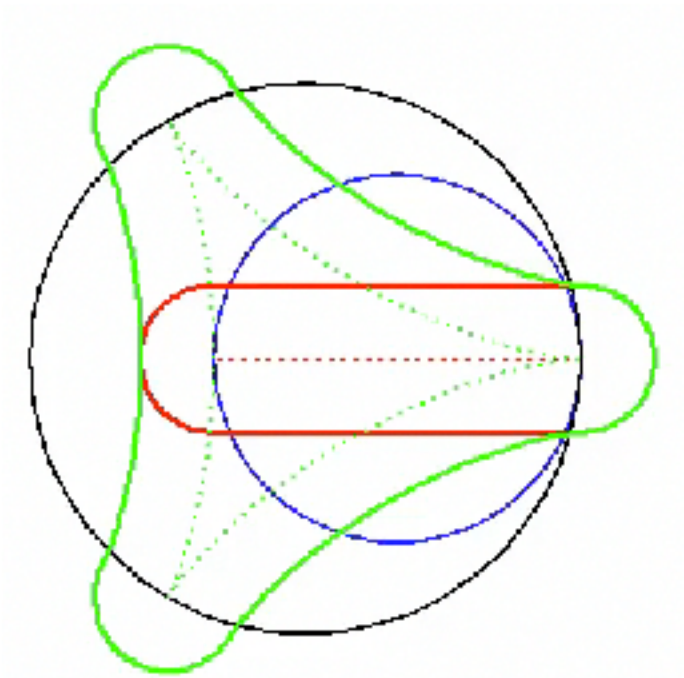
Expanding the point to a disk we get the disk moving in an excentric motion in a fixed `canal' around the diameter.





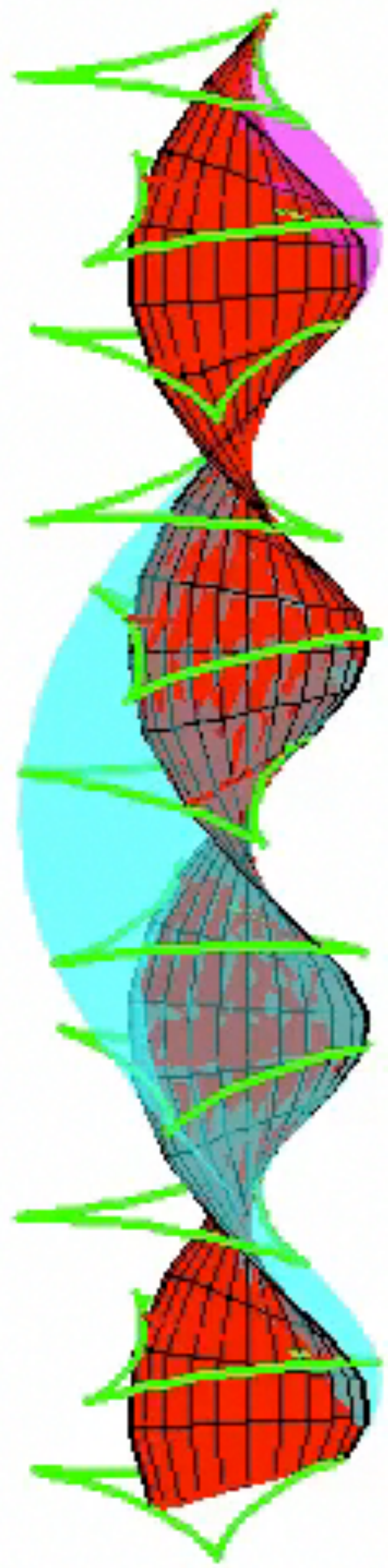
Hypo-Cycloid 3:2 construction

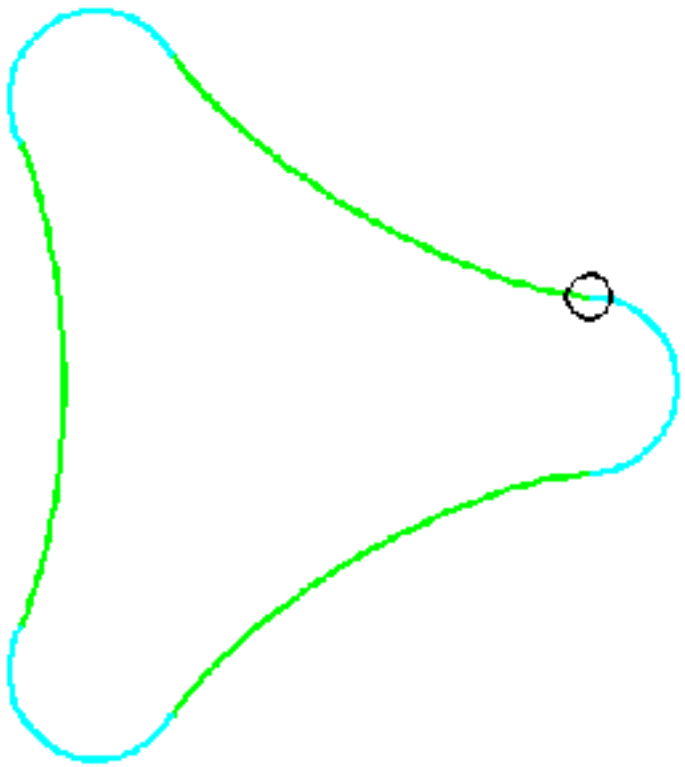


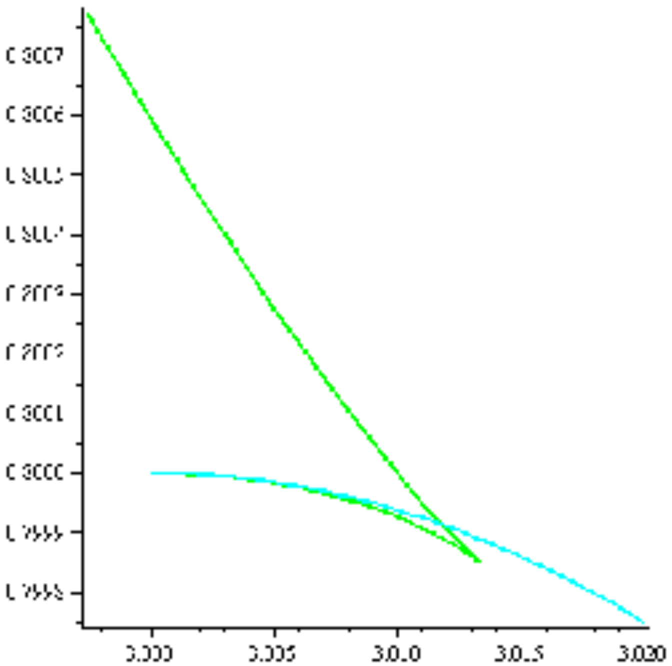


If we roll the previous construction inside a circle of radius 3, the point fixed on the smallest circle will trace out a hypocycloid.

‘fattening’ the diameter of the radius 2 circle gives us the pump chamber shape







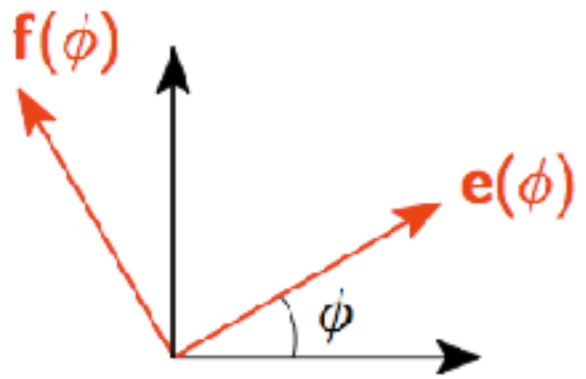
But there is a problem. The hypercycloid shape has infinite curvature at the cusps,
-- and this problem remains present in the `fat' pump chamber shape.

Grundfos asked for a mathematical description of the Moineau pump, allowing for design experimentation and a possible avoidance of singularities.

The study group provided a description in terms of the support function h(ϕ).

$$\mathbf{e}(\phi) = \mathbf{R}(\phi) \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$\mathbf{f}(\phi) = \mathbf{R}(\phi) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\phi \\ \cos\phi \end{bmatrix}$$



$(\mathbf{e}(\phi), \mathbf{f}(\phi))$ is a rotating frame.

$$\mathbf{x}(\phi) = h(\phi)\,\mathbf{e}(\phi) + h'(\phi)\mathbf{f}(\phi),$$

$$\mathbf{x}'(\phi) = (h(\phi) + h''(\phi))\mathbf{f}(\phi) = \frac{\mathrm{d}s}{\mathrm{d}\phi}\mathbf{t}.$$

So $\mathbf{f}$ is the tangent, $\mathbf{e}$ is the normal, $h = \mathbf{x} \cdot \mathbf{e}$ is the support,

$$\kappa = \frac{\mathrm{d}\phi}{\mathrm{d}s} = \frac{1}{h(\phi) + h''(\phi)} \qquad \text{is the curvature, and}$$

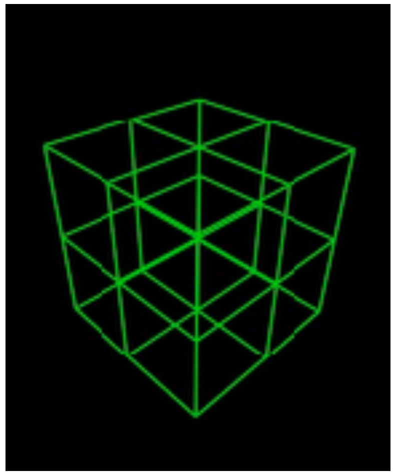
$$\rho = \frac{\mathrm{d}s}{\mathrm{d}\phi} = h(\phi) + h''(\phi) \qquad \text{is the radius of curvature.}$$

If $h + h'' \neq 0$ then the curve is regular.
Any planar curve without inflexion points can be given by its support function.

The study group proved that the singular curvature problem is unavoidable for all Moineau constructions.

The support function description also permitted the design of an optimal construction, now in use by Grundfos.

Office building freequencies (BAE Systems, Inc)



An office building has several companies placed in rooms on various floors.

All these companies have WLAN - wireless networks. Two companies with ajecent walls or floors should have WLAN running on different frequencies.

Every WLAN domain can thus be seen as a node on a graph, whose edges are the company crossing wall-and floor boundaries.

ESGI was asked to provide an estimate of the maximal number of frequencies needed; i.e. the cromatic number of the associated graph.

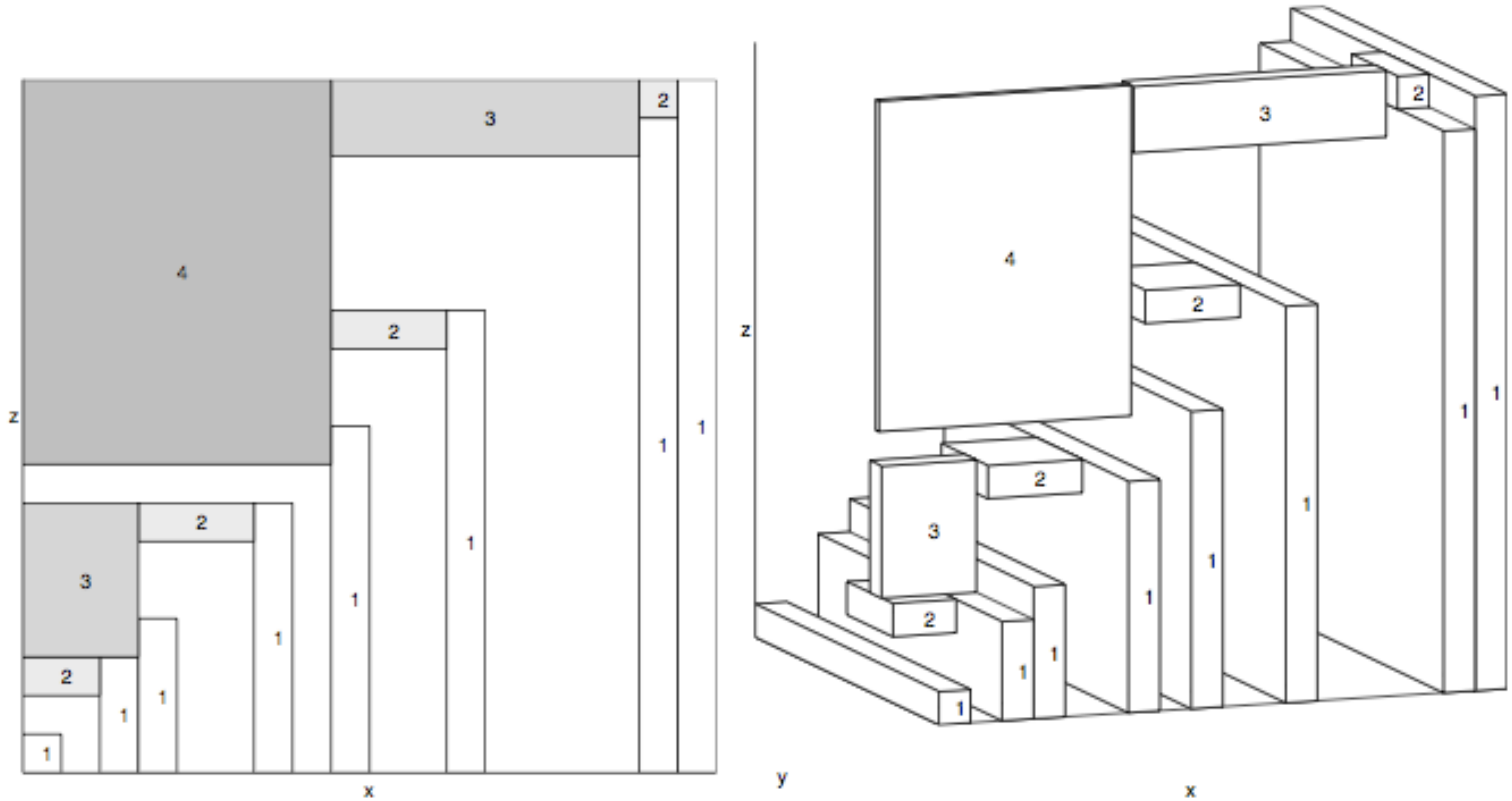


Figure 5: An arrangement of blocks in a cross-section showing some blocks up to one of Type 4 in the (x, z)-plane (left), and how they might look in 3 dimensions (right).

Hence this example shows that the chromatic number required to colour the adjacency graph of n cuboids can grow at least as fast as $\log\log n / \log\log\log n$.

Answer:

The chromatic number x(*n*) has a lower bound which increases with *n*, the number of companies. There is no upper bound for x(*n*).

In prectice, one finds relativly small values for x(n).

New (mathematical) **questions:**

How can one improve the estimate

$$\frac{\log \log n}{\log \log \log n} < x(n) < 4 \log_2(n) + 1$$

which is really wide

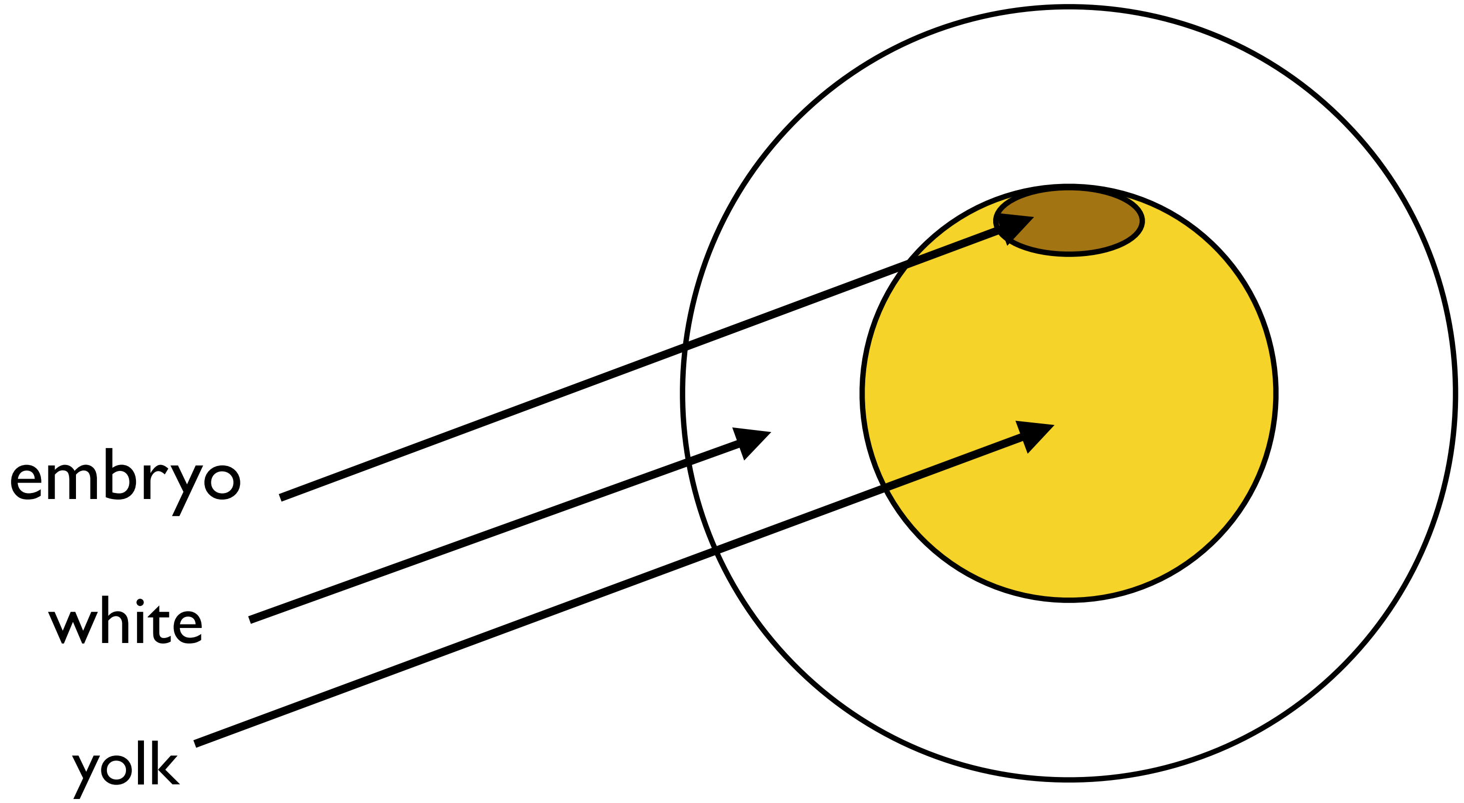
How can one find a rapid (optimal?) algorithm for coloring ?

Penguins

Spheniscus demersus.

Bristol Zoo asked for an analysis of the gravity-driven internal motions of a penguin egg.

We consider a spherical egg.



time scales

12-15 min $\approx$ 800-1000 s

Temperature diffusion ?

thermal diffusion time scale

$$\tau \sim \frac{R^2}{\kappa}$$

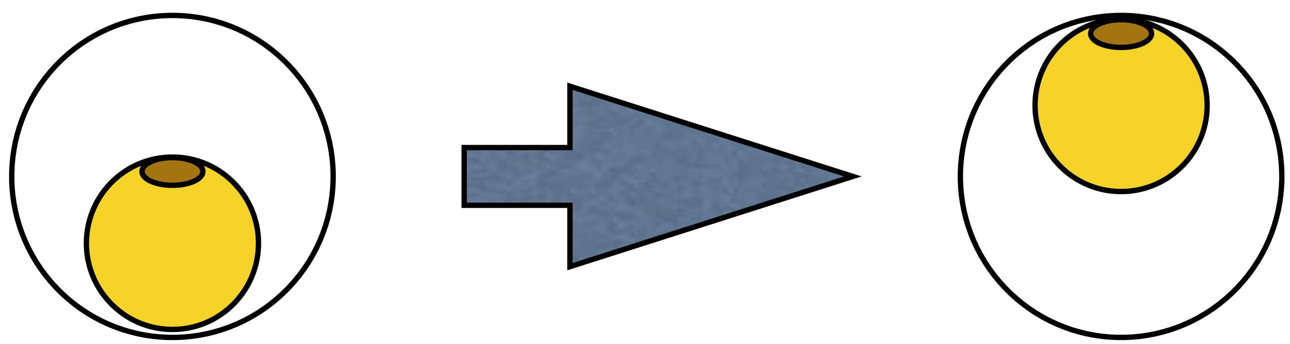
$$\frac{4 \times 10^{-4}}{1.4 \times 10^{-7}}$$

$$3 \times 10^3 \text{ s}$$

- probably not -

Buoyancy time scale ?

The yolk is lighter than the white



$$\Delta\rho \approx 0.005$$

force balance:

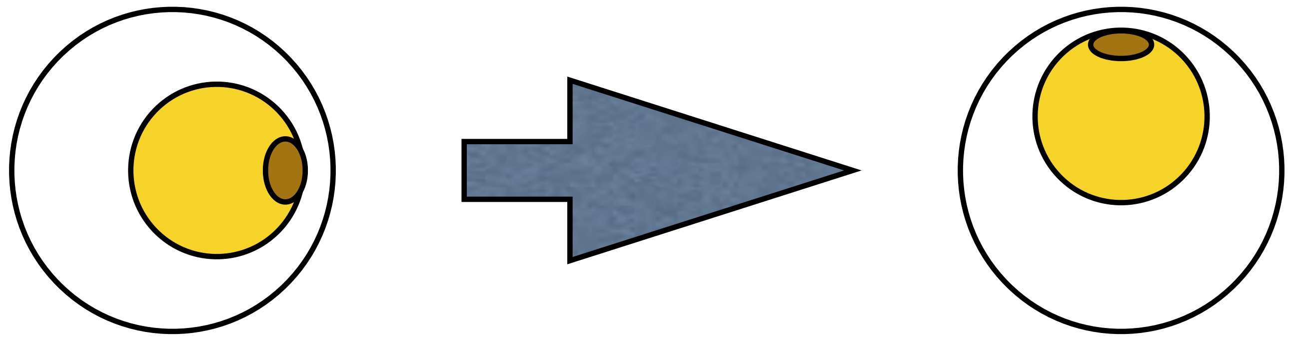
$$\frac{4\pi}{3} r_Y^3 g \Delta\rho = \frac{\mu V_y r_Y^4}{h_y}$$

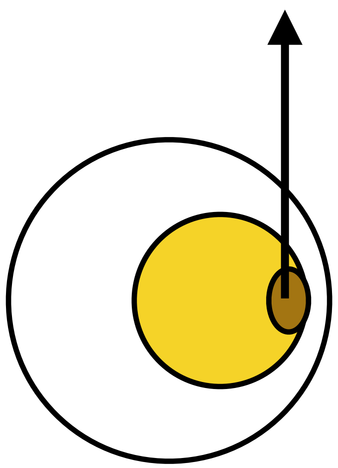
buoyancy force — friction force in thin-film motion

$$V_y \approx 5 \times 10^{-4} \frac{\text{m}}{\text{s}} \quad \Rightarrow \quad \tau = \frac{h_y}{V_y} = 4 \text{ s}$$

- probably not -

Rotational time scale ?





$$\tau_B = r_Y g \Delta m$$

buoyancy torque

$$\tau_\mu = \mu \frac{r_y \omega}{h_w}$$

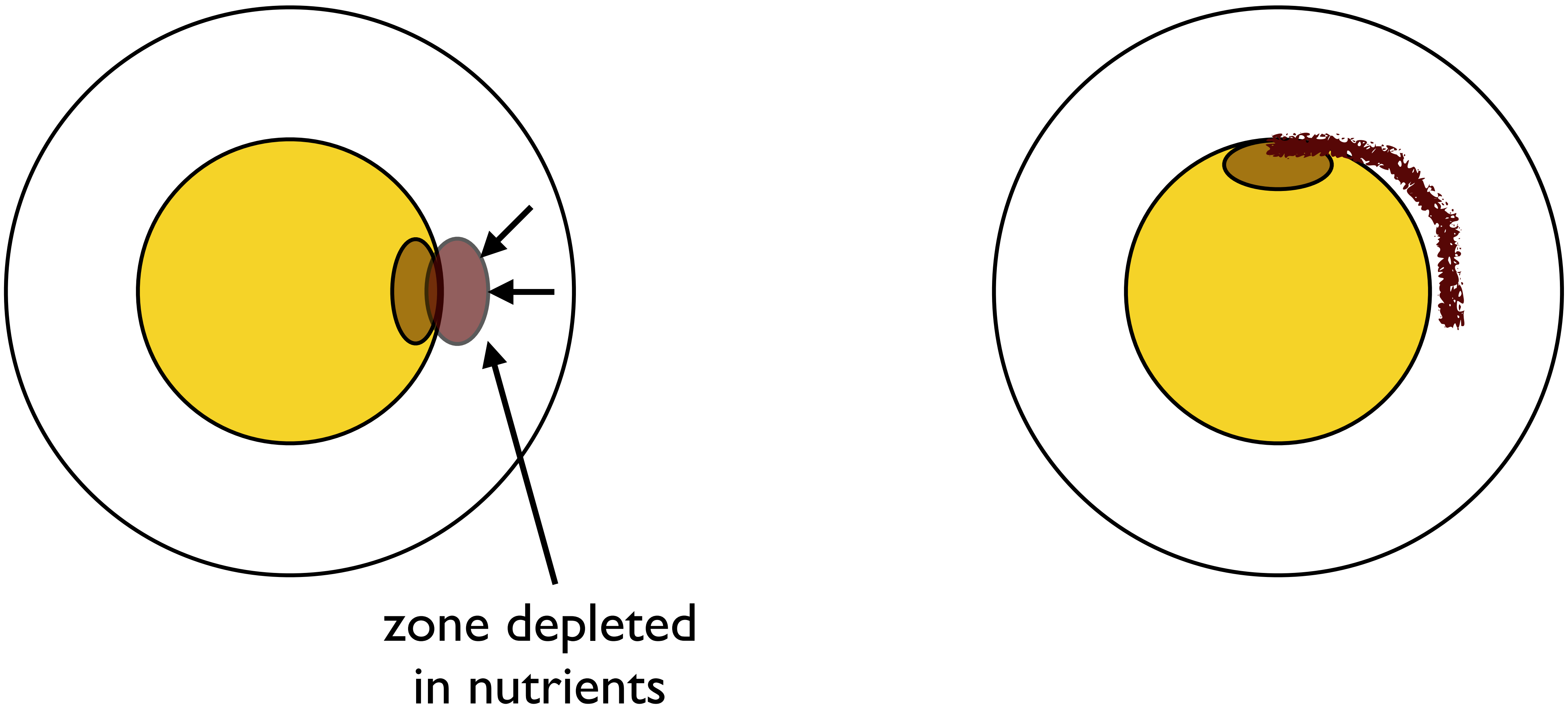
viscous torque

$$\Rightarrow \qquad \frac{1}{\omega} \approx 5 \times 10^2 \text{ s}$$

- bingo ! -

why rotate the yolk ?

shear-driven diffusion



molecular diffusitivity $D_m \approx 10^{-9} \ \frac{\mathrm{m}^2}{\mathrm{s}}$

diffusion length scale $\sqrt{D_m t} = 10^{-3} \ \mathrm{m}$

Some cracking ideas on egg incubation

Problem presented by

Duncan Bolton

Bristol Zoological Gardens

Problem summary and abstract

The preservation of rare and endangered species of birds requires finding efficient, and above all successful, methods of breeding them in captivity. One strategy adopted is to remove eggs from the mother, making her lay more eggs, and then incubating the removed eggs artificially. Of course, artificial incubation machines must attempt to replicate the conditions of natural incubation as closely as possible. Aside from careful control of

9 The role of shearing in improving the diffusion of nutrients and waste products

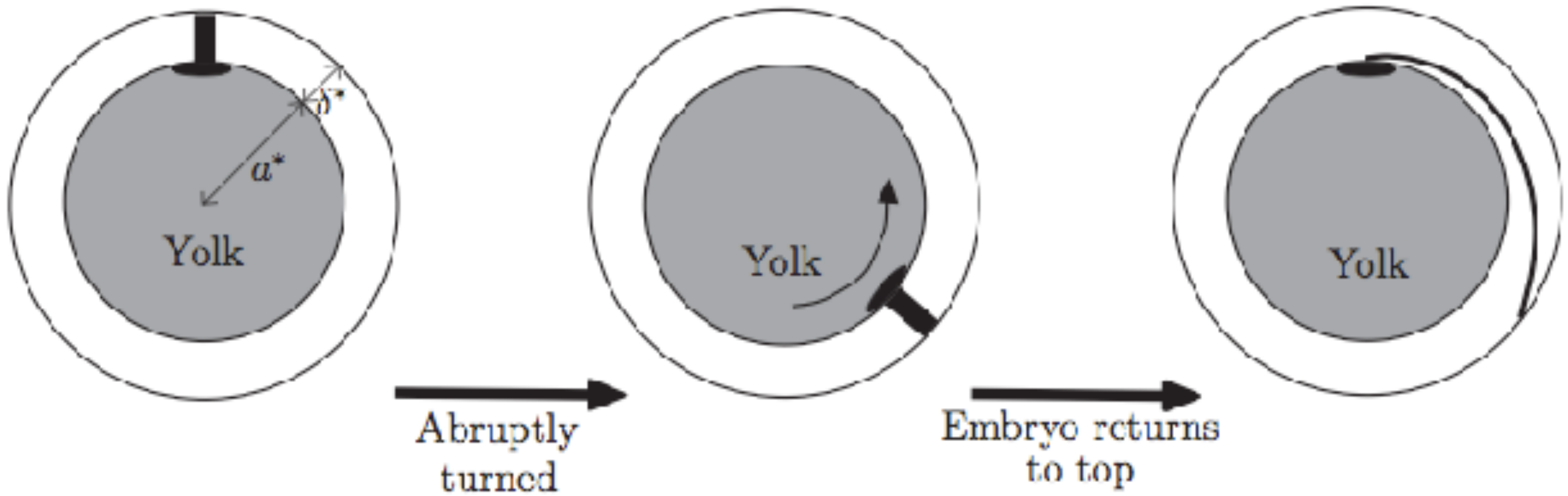


Figure 12: How a shearing flow can help to diffuse waste and nutrients

Here we attempt to explain why the turning of the egg and subsequent spinning of the yolk by the lighter embryo improves access to nutrients and disperses waste effectively. The left-hand diagram of figure 12 shows the inside of an egg with the embryo resting at the top. We assume the radius of the yolk is $a^* \sim 10^{-2}$m and that the width of the surrounding albumen layer is $\delta^* \sim 10^{-3}$m. After some time at rest, the embryo expends the nutrients in the neighbouring albumen, leaving a strip (marked in black) of albumen that has become full of waste products and devoid of nutrients. Assuming this strip to be of size $\delta^* \times \delta^*$ and the molecular diffusion coefficient to be $D \sim 10^{-9}\text{m}^2\text{s}^{-1}$ (as given in section 5.3) then the time taken to for this strip to diffuse away will be of the order

In a few cases, an analytical solution to the egg-turning model can be obtained. Two examples are presented below.

Neutrally buoyant yolk with embryo spin-up

For a neutrally buoyant yolk, $\rho_A^* = \rho_Y^*$, the yolk force equations (21) and (22) become irrelevant. The density differences must now be non-dimensionalised with $\rho_Y^* - \rho_E^*$ leading to $\Delta m = 1$ in (20). Taking the yolk to be positioned in the centre of the egg $(X, Y) = (0, 0)$, we have $h(\theta, t) = h_0$ from (18) and the squeeze-film equation (19) just leads to $\mathrm{d}P/\mathrm{d}\theta = 0$. From (20) we obtain the governing equation

$$\cos\phi(t) = \frac{2\pi\,\dot{\phi}\,(t)}{h_0}.$$

Integrating this with the initial condition $\phi = \phi_0$ at $t = 0$ leads to the solution

$$t = \frac{2\pi}{h_0}\ln\left[\frac{\sec\phi + \tan\phi}{\sec\phi_0 + \tan\phi_0}\right].$$

Note that it takes an infinite time for the embryo to get to the top, where $\phi = \pi/2$.

Buoyant yolk with no horizontal displacement and no embryo spin

In this case, we assume that the embryo stays at the top throughout, $\phi(t) = \pi/2$, and that the yolk has no horizontal displacement, $X = 0$. In other words, we are just going to examine the yolk floating to the top of the egg from some initial vertical displacement Y. From the squeeze-film equation (19), we find

$$\frac{\partial P}{\partial\theta} = \frac{12\,\dot{Y}\,(t)\cos\theta}{\left[h_0 - Y(t)\sin\theta\right]^3}.$$

Result: